Signaling and Learning in Collective Bargaining[1]

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Abstract

We analyze a two-period collective bargaining game with asymmetric information and a persistent agenda setter. Committee members (i.e., voters) have private information about their preferences over one-dimensional policy. The setter has a chance to alter the proposal if the initial one fails. When the revised proposal fails, the status-quo policy is implemented. We focus on and establish the existence of an informative equilibrium, where voters use a cut-point strategy in the initial voting. We illustrate that partial revelation of a voter' preference through the initial voting depends on three concerns: (1) his vote may be pivotal, (2) his vote provides a signal that sways the future proposal toward his ideal, and (3) his vote provides a signal to induce a more greedy proposal, which is more likely to fail. Although the third concern (i.e., *the saboteur incentive*) may dilute the incentive for preference disclosure, especially for the voter who likes the status quo the best, we show that the agenda-setting power makes the saboteur incentive dominated. Hence, an informative equilibrium exists, where the revised proposal is monotonic with respect to the vote totals. The model demonstrates the effect of the agenda-setting power on extracting information in a dynamic and collective-decision environment.

Keywords: Collective Bargaining; Asymmetric Information; Signaling

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1. Introduction and Related Literature

1.1. Introduction

The process of sequential and collective bargaining is common in politics. There are often two features in the bargaining process, especially in the U.S. system : (1) a particular politician can make formal proposals more than once on a particular issue if the initial proposal is not collectivelly approved, and (2) politicians face uncertainty about the other actors' (e.g., voters' or legislators') preferences. For example, the president can nominate cabinet secretaries or Supreme Court justices, however, the appointment must be confirmed by the United States Senate (Primo, 2002). Similarly, in the public-school budgeting process of some areas (e.g., New York State)[2] in the United States, the executive has a chance to revise the proposal if the initial one fails to gain enough support (Romer and Rosenthal, 1978, 1979). The policy proposer (i.e., agenda setter) makes proposals in pursuit of her own goal while taking into account how others respond to the proposals. Does the initial voting facilitate communication between voters and the setter? Do the vote totlas on a failed initial proposal convey credible information about voters' preferences? Does an additional chance of revising the proposal (reflected by, e.g., the term limit of an executive) magnify the agenda-setting power?

To answer these questions, we present a model of sequential and collective bargaining with asymmetric information and a persistent agenda setter. The policy is one-dimensional. The setter has a strictly increasing utility

[2] For more details, see `http://www.nyssba.org/clientuploads/nyssba_pdf/2014-school-budget-vote-timeline.pdf`.

over the policy. She faces a committee composed of a finite number of voters with spatial preferences. Each voter has private information about his ideal policy. No abstention is allowed. The setter is allowed to make an experimental proposal. The proposal is implemented and game ends if it is collectively accepted under a given voting rule. Upon the failure of the initial proposal, the setter can alter the proposal and call for a re-vote. If the revised proposal does not get enough support either, the status quo policy will be implemented.

We focus on a particular form of (symmetric) informative equilibrium, where each voter casts a positive vote in the initial period if and only if his ideal point is above an equilibrium cut-point, which depends on the initial proposal. When the initial proposal fails, based on the number of negative votes, the setter rationally infers the voters' cut-point, and Bayesian updates her belief about the voters' preferences. Given such an updated belief, the setter then chooses a revised proposal to maximize her expected utility, taking into account the fact that the voters in the last stage vote sincerely.

We illustrate that the revelation of a voter' preference in the initial voting depends on three concerns. Consistent with the literature (e.g., Meirowitz and Shotts 2009), when a voter casts his vote on the initial proposal, he not only cares about the chance when his vote is pivotal and can directly influence the policy outcome, but also takes into account the chances when the initial proposal fails and his vote as a signal can indirectly affect future proposal and therefore the policy outcome. We call the first concern *the pivot incentive*, and the latter concern *the signaling incentive*. When the initial proposal fails, a vote as a signal that sways the revised proposal toward a voter's ideal could benefit the voter if the revised proposal gets accepted. Meanwhile, because a policy proposal further away from the status quo is more likely to be rejected, in the initial voting stage a voter may also want to use a positive vote to induce a more greedy proposal, which is more likely to fail. We call these two concerns *the pro-preference incentive* and *the saboteur incentive* respectively. Hence, the signaling incentive is decomposed by these two forces.

The saboteur incentive could sometimes conflict with the other two. For example, for a voter who likes the status quo the best should cast a negative vote according to the pivot incentive and the pro-preference incentive (given the cut-point equilibrium we focus on). However, conditional on the event when a vote is not pivotal and the intitial proposal fails, the saboteur-incentive would entail a positive vote so as to induce a more extreme proposal in the second period, which will be rejected with a higher probability than in the case with a negative vote. Therefore, the saboteur-incentive may dilute the other two incentives and prevent information disclosure, thus creating a difficulty of establishing the existence of an informative cut-point equilibrium. Technically, it means that each voter's continuation payoff function endogenously violates the single-crossing condition when we guess and verify an informative cut-point equilibrium. Hence, the payoff gain of a voter in the first stage is not a monotonic function of his type (i.e., ideal point). Given an initial proposal, although we can use the fixed-point logic to get the existence of an indifferent type (who is just indifferent between casting a positive and a negative vote), we have to check the incentives of the other types of voters. Typically, a voter with an ideal point closer to the status quo policy may have a higher payoff gain, and a higher incentive to deviate from the equilibrium cut-point strategy.

In spite of the violation of the single-crossing condition, we show that the agenda-setting power makes the saboteur-incentive dominated, so that an informative cut-point equilibrium always exists. By pretending to be a supporter for an extreme reform, a voter bears a risk that his vote may be pivotal so that the first proposal gets accepted, and a risk that the more biased revised proposal will be accepted. If the setter designs the initial proposal well, she should not keep such risks at the minimum level. As the initial proposal becomes larger, at least the first risk gets magnified, so that the net benefit of misleading the setter becomes lower for the voter. A sophisticated design of the initial proposal provides appropriate risks for the voters and prevents them from deviating from the cut-point equilibrium, hence generating credible information as an efficient screening technology.

We characterize the learning effect in this informative cut-point equilibrium: more negative votes in the initial voting make the setter propose a more compromising policy when she has a chance to revise the proposal. This is because, in an informative cut-point equilibrium, conditional on the failure of the initial proposal, more negative votes in the first period make the agenda setter believe that more voters have lower demands for reform. According to the learning effect, the setter is strictly better off than in the case where she only has one chance to make the proposal.

1.2. Related Literature

This paper contributes to the literature of sequential and collective bargaining by adding private information about preferences and the learning from up-stream vote totals. Models of collective bargaining with complete information have been well developed (Baron and Ferejohn, 1989; Banks and Duggan, 2000, 2006; Eraslan, 2002; Kalandrakis, 2004; Battaglini and Coate, 2007; Duggan and Kalandrakis, 2012; Nunnari, 2012; Anesi and Seidmann, 2014; Nunnari and Zápal, 2013; Jeon, 2015a,b). However, very few papers analyze sequential and collective bargaining with asymmetric information. Our model is among the first that achieves this goal. Different from Baron and Ferejohn (1989), and in line with Romer and Rosenthal (1978, 1979), Diermeier and Fong (2011, 2012), and Dahm and Glazer (2013), we assume that an authority exists who persistently holds agenda-control power checked by the requirement of collective approval.

Matthews (1989) studies how cheap talk reveals information of the veto player's preference before one-shot bargaining. Built on this, Chen and Eraslan (2013, 2014) study three-player legislative bargaining with two-dimensional policy (ideology and redistribution transfer) under simple majority rule before which cheap talk is allowed. They find that two voters under simple majority may make the setter worse off than in the case with just a single voter having an absolute veto power. Meirowitz (2007) studies how information can be fully revealed through cheap-talk communication before collective bargaining with a common value setup. Agranov and Tergiman (2014a,b) and Baranski and Kagel (2013) use laboratory experiments to address the effect of communication in the Baron-Ferejohn environment. Different from those models with cheap talk, in our framework, voters' private information about their preferences can be revealed by the vote totals over the initial proposal. It is a costly signaling process instead of the costless cheap-talk communication. Tsai and Yang (2010) study a majoritarian bargaining model where players have private information about their discount factors. In our model, the uncertainty is about the voters' spatial preferences (i.e., ideal points) rather than the discount factor. Furthermore, we allow interaction among arbitrary number of voters instead of three voters.

Our model is also related to recent work on how up-stream actions may transmit information and shape the down-stream collective actions. For example, Piketty (2000), Razin (2003), Iaryczower (2008), Bond and Eraslan (2010) study the information transmission in a partially common-value voting setup. Messner and Polborn (2012) study voters' learning about their own preferences in an environment with sequentially collective decision making. In private-value environments, Shotts (2006) and Meirowitz and Shotts (2009) study how voters use the upstream election to signal their preferences and to indirectly affect future policy. In Meirowitz and Shotts (2009), competition between the two parties make their second-period proposals converge. However in our model, monopoly agenda control makes the political power asymmetrically distributed. Each voter faces uncertainty and forms heterogeneous beliefs about the second-period policy that will be implemented. It is the monopoly agenda control subject to collective approval together with the preference uncertainty that creates the unique saboteur incentives in the initial voting stage of our model.

The remainder of the paper is organized as follows. We introduce the basic setup in Section 2. We then use guess-and-verify method to establish a cut-point equilibrium. Specifically, in Section 3, we provide some preliminary results, and show necessary conditions of such an informative-voting equilibrium. In Section 4, we characterize voters' incentives of voting in the initial stage, and show that each voter's payoff gain is not a monotonic function of his ideal point. In Section 5, we derive our main result about the existence of an informative-voting equilibrium, shedding light on how the initial agenda-setting power generates credible information. In Section 6, we discuss our results and possible extensions.

2. Model Setup

2.1. Institutional Arrangement

We consider a committee with N voters and one agenda setter. The one dimensional proposal will be collectively approved if and only if at least q voters cast positive votes. Otherwise the proposal will be rejected. We refer to the policy as the *budget*, however it can be any public policy in general. $N \geq q \geq 2$ and $q, N \in \mathbb{Z}^+$. We normalize the status quo s to 0. The policy space is $[0, +\infty)$.

The **timing** of the game is as follows:

In Period 1 The agenda setter makes a policy proposal b_1. Then voters simultaneously decide whether to accept it. If it is accepted, b_1 is implemented and game ends.

In Period 2 If the first proposal is rejected, the setter makes a new one b_2. If the revised proposal is rejected too, the status quo s will be implemented. Otherwise b_2 is implemented.

2.2. Preferences

Voter i's utility over policy x is

$$u_v(x; \theta_i) = -2\theta_i x + x^2, \tag{1}$$

with ideal point θ_i.[3]

Similarly as in Romer and Rosenthal (1978, 1979), Banks (1990, 1993), Diermeier and Fong (2012), and Anesi and Seidmann (2014), we assume that the setter wants to maximize the budget and has a monotone utility over policy x.

$$u_A(x) = x. \tag{2}$$

In the Appendix, we prove the results based on generalized preferences: $u_v(x;\theta_i) = \theta_i v(x) - C(x), u_A(x) = \sigma v(x)$ (where $\sigma > 0$) providing that certain technical conditions are satisfied. We assume that players put equal weights on the two periods.[4]

2.3. Information Structure

The private ideal point θ_i of each voter is i.i.d. drawn from an absolutely continuous distribution $F(\theta)$ with support $[\underline{\theta}, +\infty)$, where $\underline{\theta} \leq 0$.

The distribution from which voters' ideal points are drawn is common knowledge. In addition, each voter privately observes his own ideal point.

The setter's preference is public information. We make the following assumption about the distribution function $F(\theta)$.

Assumption 1. *Committee Composition*

(1.1) $F(\cdot)$ *is twice continuously differentiable on* $[\underline{\theta}, +\infty)$; *(1.2) the probability density function* $f(\theta) > 0, \forall \theta \in [\underline{\theta}, +\infty)$; *and (1.3) the hazard rate*

[3] This assumption helps to simplify some notations especially when the status quo $s = 0$. It is equivalent to assume that $u_i(x;\theta_i) = -(\theta_i - x)^2$.

[4] Our result does not depend on the discount factor of the second period. As voters sufficiently discount future payoffs, their first period voting behavior converges to the sincere voting, where voters simply compare their payoffs under the initial proposal and under the status quo.

$\frac{f(\theta)}{1-F(\theta)}$ *is weakly increasing for* $\theta \in [\underline{\theta}, +\infty)$.[5]

The increasing hazard-rate condition in Assumption *1* is a standard condition in the Bayesian Game literature (Banks, 1993). Many distributions satisfy this assumption, such as exponential distribution and normal distribution (Bergstrom and Bagnoli, 2005).

2.4. Strategies and Equilibrium

In Period 1, the setter chooses an initial proposal $b_1 \in [0, +\infty)$. In the voting stage of the second period, we assume that voters vote sincerely. This assumption excludes the weakly dominated strategies in the final stage. Thus each voter votes for the revised proposal against the status quo if and only if $\theta_i \geq \frac{1}{2}b_2$. Formally we use $V^2(\theta_i, b_2)$ to denote the last stage voting strategy, hence we have:

$$V^2(\theta_i, b_2) = \begin{cases} 1 & \text{if } \theta_i \geq \frac{1}{2}b_2 \\ 0 & \text{if } \theta_i < \frac{1}{2}b_2 \end{cases}. \tag{3}$$

We focus on symmetric *Perfect Bayesian cut-point equilibrium*, i.e., an equilibrium in which each voter casts a positive vote in the first period if and only if his ideal policy is greater or equal to the cut-point $k^*(b_1)$, which depends on the initial proposal b_1. We use $r \in \{0, ..., N - q + 1, ...N\}$ to denote the number of negative votes in the first period. The **equilibrium** $\{b_1^*$,

[5]The increasing-hazard rate assumption does not imply unimodality, and we do not assume that the voters' ideal points are sufficiently concentrated. Hence, our result that the violation of the single-crossing condition does not destroy the existence of an informative equilibrium, is not due to the ability of the setter to place the proposal "close" to most voters.

$k^*(b_1)$, $P^*(r, b_1)|_{r\in\{N-q+1,...,N\}}$, $b^*(r, b_1)|_{r\in\{N-q+1,...,N\}}$, $V^2(\theta_i, b_2)\}$ involves the following requirements:

(0) given voters' equilibrium voting strategies $k^*(\cdot)$ and $V^2(\theta_i, b_2)$, as well as the revising-proposal strategy $b^*(r, b_1)$, b_1^* should maximize the setter's expected payoff;

for any $b_1 \in [0, +\infty)$,

(1) given the other voter's equilibrium cut-point strategy $k^*(b_1)$, the second-period equilibrium proposing strategy $b^*(r, b_1)$, and the last stage sincere voting strategy $V^2(\theta_i, b_2)$, voter i with any type θ_i does not have an incentive to deviate from the cut-point strategy $k^*(b_1)$ in the first period voting;

(2) $P^*(r, b_1)$ represents the setter's belief about the voters' ideal points given r negative votes, the initial proposal b_1, and her rational expectation of $k^*(b_1)$; it is derived by Bayes' rule whenever possible;

(3) the equilibrium proposal $b^*(r, b_1)$ needs to maximize the setter's welfare given her equilibrium belief about voters' ideal points $P^*(r, b_1)$; and

(4) $V^2(\theta_i, b_2)$ is the last stage sincere voting strategy, and is determined by equation (3).

For convenience, we make the following definitions.

Definition 1. *For any $b_1 \in [0, +\infty)$, we call $\{k^*(b_1)$, $P^*(r, b_1)|_{r\in\{N-q+1,...,N\}}$, $b^*(r, b_1)|_{r\in\{N-q+1,...,N\}}$ and $V^2(\theta_i, b_2)\}$ a **subgame**[6] **equilibrium** if they satisfy conditions (1)-(4). We also call $k^*(b_1)$ a **subgame-equilibrium cut-point** if $k^*(b_1)$ and some other strategies form a subgame equilibrium.*

[6] In this subgame, nature starts first to determine the voters' preferences.

We also make a refinement of equilibria.

Refinement 1. *We focus on equilibria where no matter what (other than the status quo) the setter proposes, none of the voters accepts the proposal with probability 1, i.e.,* $k^*(b_1) > \underline{\theta}$ *whenever* $b_1 > 0$.

Lemma 1. *(1) With Refinement 1, for a given* $b_1 > 0$*, any subgame-equilibrium cut-point must satisfy* $k^*(b_1) > 0$ *(including* $k^*(b_1) = +\infty$*);*

(2) when $b_1 = 0$*, the setter's expected payoff in any subgame equilibrium with* $k^*(b_1) \in [\underline{\theta}, 0]$ *is 0, weakly less than her equilibrium expected payoff in the static game when she has only one chance to make the proposal (which is equivalent to the case when* $k^*(\cdot) = +\infty$*).*

According to the lemma, without loss of generality, we now only need to focus on the subgame-equilibrium with cutpoint $k^*(b_1) > 0$ (including $k^*(b_1) = +\infty$). Generally we are interested in an equilibrium with an interior subgame-equilibrium cut-point $k^*(b_1^*) \in (0, +\infty)$ under the optimal initial proposal b_1^*. We call it an **informative-voting equilibrium**. We are interested in and will characterize the necessary conditions and the existence of such an equilibrium. The first step is to see what kind of subgame-equilibrium cut-point $k^*(b_1)$ a given initial proposal $b_1 \geq 0$ can induce. If b_1 induces a subgame equilibrium with $k^*(b_1) \in (0, +\infty)$, we call it a *subgame informative-voting equilibrium*.

3. Preliminary Results: Setter's Belief and the Second-Period Proposal

In the second period, when the agenda setter makes her best proposal, she only needs to target the "pivotal" ideal point, which we will specify in the next paragraph. Without loss of generality, we assume that $P^*(y, b_1)$ reflects the information only about the distribution of the "pivotal" ideal point (given voters' subgame-equilibrium cut-point $k_B^*(b_1)$).

If $N - r \geq q$, the initial proposal will be passed; otherwise the setter observes $r \geqslant N - q + 1$ negative votes, and updates her belief about the "pivotal" voter's ideal policy. Specifically she will target the $(N - q + 1)$th smallest ideal point among the r i.i.d. draws with the distribution $\widetilde{F}(t_i; k) \triangleq \Pr(\theta_i \leqslant t_i \mid \theta_i < k)$ (which is $\frac{F(t_i)}{F(k)}$ when $t_i \in [\underline{\theta}, k]$). In other words, the targeted distribution is $\Omega(t_i|r; k) = \widetilde{F}_{r,N-q+1}(t_i; k)$, which is the distribution of the $N - q + 1$ th lowest order statistics from the r i.i.d. random variables $\theta_i \mid_{\theta_i < k}$.

By the construction of the function $\Omega(\cdot|r; k)$, the equilibrium beliefs in a subgame informative-voting equilibria can be directly recovered given the subgame-equilibrium cut-point $k^*(b_1) > 0$:

$$P_B^*(r, b_1) = \{\Omega(\cdot|r; k^*(b_1))\}, \text{ for } r \geqslant N - q + 1 \text{ and } b_1 \in [0, +\infty).^7 \qquad (4)$$

We now derive the agenda setter's best response to the total number of negative votes r and her belief about the voters' cut-point k. Suppose

[7] If $k_B^*(b_1) = +\infty$, we have $P_B^*(N, b_1) = \{\Omega(\cdot|N; k_B^*(b_1) = +\infty)\}$, and $P_B^*(r, b_1)|_{r<N}$ is not well defined on the equilibrium path.

the revised proposal is b_2. With probability $[1-\Omega(\frac{1}{2}b_2|r;k)]$, the revised proposal will be accepted, so that the setter gets $u_A(b_2)$; with probability $\Omega(\frac{1}{2}b_2|r;k)$, the revised proposal fails and the setter gets a payoff $u_A(0)=0$. $[1-\Omega(\frac{1}{2}b_2|r;k)]u_A(b_2)$ is thus the setter's expected payoff. As the setter increases her proposal b_2, she gets a higher payoff if the proposal is accepted because $u_A(b_2)$ is increasing in b_2. However a less compromising proposal will be more likely to be rejected since $\Omega(\frac{1}{2}b_2|r;k)$ is increasing in b_2. Hence, her optimal proposal entails such a tradeoff, and is determined by

$$\beta(r;k) \triangleq \underset{b_2\in[0,+\infty)}{\arg\max}[1-\Omega(\frac{1}{2}b_2|r;k)]u_A(b_2), \text{ for } r \geqslant N-q+1. \tag{5}$$

By definition, the setter's equilibrium revised proposal $b^*(r,b_1)$ (when the initial proposal is b_1 and there are r negative votes in the first period) should be equal to the best response $\beta(r;k)$ evaluated at the same number of negative votes r, and the subgame-equilibrium cut-point $k^*(b_1)$ induced by b_1:

$$b^*(r,b_1) \in \beta(r;k^*(b_1)), \text{ for } r \geqslant N-q+1 \text{ and } b_1 \in [0,+\infty). \tag{6}$$

Lemma 2 characterizes the properties of $\beta(r;k)$.

Lemma 2. $\beta(r;k)$ *defined in equation (5) represents the setter's optimal revised proposal(s) when she faces* $r \geqslant N-q+1$ *negative votes, and believes that voters use a symmetric cut-point strategy with the cut-point* $k \in (0,\overline{\theta})$. *We have:*

(1) $\beta(r;k)$ *is single-valued, so that* $b^*(r,b_1)=\beta(r;k^*(b_1))$, *provided* $k^*(b_1) > 0$ *is a subgame-equilibrium cut-point induced by* $b_1 \in [0,+\infty)$, *and* $r \geqslant N-q+1$;

(2) $\beta(r;k)$ is continuously differentiable and strictly increasing in k;

(3) $k > \frac{1}{2}\beta(r;k)$, so that on the equilibrium path, the one who casts a positive vote in the first period also approves the equilibrium revised proposals; and

(4) $\beta(r;k)$ is strictly decreasing in the total negative votes, r.

(We prove a more comprehensive version, Lemma 8 in the Appendix.)

The first part of the lemma suggests how we can use the functions $\beta(r;k)$ and $k^*(b_1)$ to recover the equilibrium revised proposal $b^*(r, b_1)$ given r negative votes and the initial proposal b_1. The second part of the lemma is a technical preparation for establishing the existence of an informative equilibrium. As the upper bound of the distribution k moves upward, given the same number of negative votes r, the setter's belief about the voters' policy demands becomes higher, therefore she will make a proposal that is further away from the status quo. The third part of the proposition suggests that on the equilibrium path, the one who casts a positive vote in the first period also approves the equilibrium revised proposals. This observation will help us to simplify the expression of the continuation payoffs in the following section. The last part of the lemma suggests that a larger number of negative votes in the first period drives the setter to make a more compromising proposal in the second period. This reflects the signaling feature of such an informative equilibrium (if any): voters can use their votes to signal their preferences and influence the setter's proposal. It is a necessary condition of *informative-voting equilibria,* as specified in the following corollary.

Corollary 1 (A Necessary Condition of Informative-Voting Equilibria). *In any informative-voting equilibrium (i.e., an equilibrium with* $k^*(b_1^*) \in (0, +\infty)$ *), the revised proposal* $b^*(r, b_1^*)$ *is strictly decreasing in the total number of negative votes* r*, provided* $r \geqslant N - q + 1$.

If an informative-voting equilibrium exists, on the equilibrium path, upon receiving more negative votes from the voters, the setter believes that the voters' ideal points are closer (in the probability sense) to the status quo, therefore will try a more moderate reform proposal.

4. Voting Incentives in the First Period

Given any initial proposal b_1, we will characterize each voter's incentive to vote in the first period. Specifically we will pin down each voter's utility difference between voting yes and no in the first period. We call it a *payoff gain* thereafter. When calculating the continuation payoffs, we take into account their equilibrium voting strategies in the last stage $V^2(\theta_i, b_2)$, and the setter's best response to the vote totals $\beta(r; k)$. Similar to Meirowitz and Shotts (2009), the payoff gain is calculated at two classes of events: the event when there are exactly $q-1$ positive votes among others and the voter is pivotal, and the events when there are less than $q-1$ positive votes among others.[8] We call the payoff gain calculated in the pivot event the *pivot* payoff gain, and the payoff gain calculated in the latter events the *signaling* payoff gain.

[8] When there are more than $q-1$ positive votes among others, the payoff gain is always 0 since the initial proposal gets passed and the game ends.

4.1. Pivot Incentives

Given the other voters' cut-point k, a voter will be pivotal with probability $\binom{N-1}{q-1}F(k)^{N-q}(1-F(k))^{q-1}$. In this event, he can directly choose whether to accept the initial proposal b_1 or not. If he casts a positive vote, he gets $u_v(b_1;\theta_i)$. If he casts a negative vote, he gets $V(\theta_i, \beta(N-q+1;k), N-q, k)$, where $V(\theta_i; b_2, x, k)$ is the second-period continuation payoff of a voter with ideal point θ_i when he observes x negative votes among others. Specifically, in the pivot event, the voter knows that exactly $(q-1)$ voters among others have ideal points above the cut-point k. According to part (3) of Lemma 2, those $q-1$ voters will always approve the setter's equilibrium revised proposal b_2. As long as $\theta_i \geq \frac{1}{2}b_2$, voter θ_i will also cast a positive vote in the second period, therefore b_2 will be accepted and a voter with ideal point θ_i gets $u_v(b_2;\theta_i)$. When $\theta_i < \frac{1}{2}b_2$, whether b_2 will be accepted or not depends on the maximum ideal point among the $(N-q)$ voters who cast negative votes in the first period. Therefore,

$$V(\theta_i; b_2, N-q, k) = \begin{cases} u_v(b_2;\theta_i) & \text{if } \theta_i \geq \frac{1}{2}b_2 \\ [1-\widetilde{F}_{N-q,N-q}(\frac{1}{2}b_2;k)]u_v(b_2;\theta_i) & \text{if } \theta_i < \frac{1}{2}b_2 \end{cases}.^{9} \quad (7)$$

Thus, the pivot payoff gain is $V_{piv}(\theta_i;k,b_1) = \binom{N-1}{q-1}F(k)^{N-q}(1-F(k))^{q-1}\widetilde{V}_{piv}(\theta_i;k,b_1)$, where

$$\widetilde{V}_{piv}(\theta_i;k,b_1) = u_v(b_1;\theta_i) - V(\theta_i, \beta(N-q+1;k), N-q, k). \quad (8)$$

[9] For notational convenience, define $\widetilde{F}_{r,0} = 1$, for any $r = 0, 1, ...N$.

4.2. Signaling Incentives

When a vote is not pivotal and the initial proposal gets rejected, this non-pivotal vote serves as a signal. Voter i can indirectly change future proposal by contributing to the total negative votes. Suppose there are exactly j ($\leq q-2$) positive votes among the other voters, If he votes yes, the total negative votes will not change, thus the revised proposal is $\beta(N-1-j;k)$. If he votes no, the total negative votes will be $N-j$ so that the revised proposal is $\beta(N-j,k)$. Define $P_{2j} \triangleq \binom{N-1}{j} F^{N-1-j}(k)[1-F(k)]^j$, which is the probability that there are exactly j positive votes among the other voters. The signaling payoff gain is $V_{sig}(\theta_i;k) = \sum_{j=0}^{q-2} P_{2j}\widetilde{V}^j_{sig}(\theta_i;k)$, where

$$\widetilde{V}^j_{sig}(\theta_i;k) \triangleq V(\theta_i;\beta(N-1-j;k),N-1-j,k) - V(\theta_i;\beta(N-j;k),N-1-j,k). \tag{9}$$

$V(\theta_i;b_2,N-1-j,k)$ is the second-period continuation payoff of a voter with ideal point θ_i when he observes $(N-1-j)$ negative votes among others. Notice that voters with different ideal points may have different perceptions about the chance that a certain revised proposal will be accepted. When $\theta_i \geq \frac{1}{2}b_2$, the voter θ_i will accept b_2 so that b_2 will be passed if and only if there are at least $q-1$ positive votes from others. From the perspective of the voter θ_i, the "pivotal" voter is the one whose ideal point is the $(N+1-q)$th smallest one among the $N-1-j$ ideal points $\theta_j|_{\theta_j<k}$. When $\theta_i < \frac{1}{2}b_2$, the voter θ_i will reject b_2 so that b_2 will be passed if and only if there are at least q positive votes from others. From the perspective of the voter θ_i, the "pivotal" voter is the one whose ideal point is the $(N-q)$th smallest one among the $N-1-j$ ideal points $\theta_j|_{\theta_j<k}$. As a result, we have

$$V(\theta_i; b_2, N-1-j, k) = \begin{cases} [1-\widetilde{F}_{N-1-j,N+1-q}(\frac{1}{2}b_2; k)]u_v(b_2;\theta_i) & \text{if } \theta_i \geq \frac{1}{2}b_2 \\ [1-\widetilde{F}_{N-1-j,N-q}(\frac{1}{2}b_2; k)]u_v(b_2;\theta_i) & \text{if } \theta_i < \frac{1}{2}b_2 \end{cases}. \tag{10}$$

4.3. Voting Incentives

The (total) payoff gain is the sum of the pivot effect and the signaling effect:

$$V_{diff}(\theta_i; k, b_1) = V_{piv}(\theta_i; k, b_1) + V_{sig}(\theta_i; k). \tag{11}$$

By checking the continuity of the continuation value functions, we know that the payoff-gain function is continuous.

Lemma 3. *$V_{diff}(\theta_i; k, b_1)$ is continuous in (θ_i, k, b_1) on $[\underline{\theta}, +\infty) \times (0, +\infty) \times [0, +\infty)$.*

Based on the payoff-gain function, we can restate the conditions of the equilibrium. Given $b_1 \in [0, +\infty)$, $k^*(b_1) > 0$ is a subgame-equilibrium cut-point if and only if the following conditions are satisfied:

(i) $V_{diff}(k^*(b_1); k^*(b_1), b_1) = 0$; (ii) $V_{diff}(\theta_i; k^*(b_1), b_1) \geq 0$, whenever $\theta_i \geq k^*(b_1)$; (iii) $V_{diff}(\theta_i; k^*(b_1), b_1) \leq 0$, whenever $\theta_i < k^*(b_1)$.

According to the above conditions, we can verify that, for any initial proposal $b_1 \in [0, +\infty)$, a pooling strategy $k^*(b_1) = +\infty$ can always be supported as a subgame-equilibrium cut-point. Putting $k = +\infty$ into the payoff-gain function, we get

$$V_{diff}(\theta_i; k = +\infty, b_1) = V(\theta_i; \beta(N-1, +\infty), N-1, +\infty) - V(\theta_i; \beta(N, +\infty), N-1, +\infty). \tag{12}$$

In fact, $\beta(N, +\infty)$ is exactly the policy the setter will propose in the static game without the initial period. However $\beta(N-1, +\infty)$ is not well defined on the equilibrium path. It is the policy that the setter proposes when she observes $(N-1)$ negative votes while she knows that all voters vote no. It depends on the off-the-equilibrium-path beliefs. As long as we appropriately assign the off-the-equilibrium-path beliefs such that $\beta(N-1, +\infty) = \beta(N, +\infty)$ (i.e., given all types of voters vote no in the first period, the setter holds the same belief when she receives only $(N-1)$ negative votes as when she receives a unanimous rejection), the incentive compatibility constraint becomes trivial and always holds. As a result, we have

Lemma 4. *(A pooling equilibrium always exists.) For any initial proposal* $b_1 \in [0, +\infty)$, *there always exists a subgame equilibrium with* $k^*(b_1) = +\infty$. *In this subgame equilibrium, all types of voters vote against the first proposal and no information is disclosed before the second period. In addition, the setter holds the same belief when she receives only* $(N-1)$ *negative votes as her belief on the equilibrium path.*

In the following, we will check the existence of a *subgame informative-voting equilibrium* $k^*(b_1) \in (0, +\infty)$ for a fixed initial proposal $b_1 \in (0, +\infty)$.

4.4. Non-monotonicity of Voters' Payoff-Gain Functions

Given the initial proposal $b_1 \in (0, +\infty)$, if a subgame equilibrium with $k^*(b_1) \in (0, +\infty)$ exists, a necessary condition is that the type $\theta_i = k^*(b_1)$ is always indifferent between voting yes and no, i.e., $V_{diff}(\theta_i = k^*(b_1); k^*(b_1), b_1) = 0$. The following lemma shows that such an indifferent type always exists.

Lemma 5. *For $b_1 \in (0, +\infty)$, the set $\widehat{K}(b_1) \triangleq \{k \in (0, +\infty) : V_{diff}(\theta_i = k; k, b_1) = 0\}$ is not empty.*

(See Appendix for the proof.)

To investigate the incentive compatibility constraints of the other types, we fully characterize the functions $\widetilde{V}_{piv}(\theta_i; k, b_1)$ and $\widetilde{V}^j_{sig}(\theta_i; k)$, in Equations (A6) and (A8) of the Appendix. In many Bayesian games with continuum types, the existence of an indifferent type (i.e., a local condition) is equivalent to the existence of a cut-point equilibrium, because the payoff-gain function is exogenously monotonic. However, as shown by the following lemma, the endogenous payoff-gain function of each voter may not be monotonic.

The intuition is that in the signaling channel, there are two effects. By sending a signal that is consistent with the preference, a voter can sway the revised proposal toward his own ideal. On the other hand, by sending a signal against the status quo will make the revised proposal more extreme, therefore more likely to fail. The second effect can dilute the incentive of disclosing the true preference, especially for the voter who likes the status quo the best. Formally we have the following lemma.

Lemma 6 (The Saboteur-Incentive in Signaling). *(1) For $j = 0, 1, ...(q-2)$, $\widetilde{V}^j_{sig}(\theta_i; k)$ is strictly increasing in θ_i when $\theta_i \geq \frac{1}{2}\beta(N - j - 1; k)$, and* *strictly decreasing when* $\theta_i < \frac{1}{2}\beta(N-j-1;k)$*, except for the case when* $q = N$*, we have* $\widetilde{V}^j_{sig}(\theta_i;k) \equiv 0$ *when* $\theta_i < \frac{1}{2}\beta(N-j;k)$*; (See Figure 2.)*

(2) $\widetilde{V}_{sig}(\theta_i;k)$ *is strictly increasing in* θ_i*, when* $\theta_i > \frac{\beta(N-q+1;k)}{2}$*;* $\widetilde{V}_{sig}(\theta_i;k)$ *is strictly decreasing in* θ_i*, when* $\theta_i < \frac{\beta(N;k)}{2}$*, except for the case when* $q = N$*, we have* $\widetilde{V}_{sig}(\theta_i;k) \equiv 0$ *when* $\theta_i < \frac{1}{2}\frac{\beta(N;k)}{2}$*;*

(3) $\widetilde{V}_{piv}(\theta_i;k,b_1)$ *is strictly increasing in* θ_i*, provided* $b_1 > \beta(N-q+1;k = +\infty)$*. (See Figure 1.)*

(See Appendix for the proof of Lemma 10.)

Part (3) of Lemma 6 suggests that the incentive in the pivot effect is well shaped, i.e., strictly increasing, provided the initial proposal is sufficiently large (shown in Figure *1*). However the signaling incentive is not monotonic. By Part (1) of Lemma 10, each piece $\widetilde{V}^j_{sig}(\theta_i;k)$ in the signaling effect is quasi-convex in θ_i, but not convex (shown in Figure *2*). The non-monotonicity with respect to θ_i creates a difficulty of establishing the existence of an informative cut-point equilibrium: a type of voter who is closer to the status quo may have a higher incentive to approve the initial proposal. If a voter with an ideal point $\theta_i < k^*(b_1)$ deviates from the equilibrium stratey and casts a positive vote instead, the setter will receive one additional positive vote, and believe that the voters demand a more extreme policy than what they actually demand. However, a more extreme proposal in the second period will be rejected with a higher probability. Once it gets rejected, the voter will enjoy the benefit of the status quo policy. The non-monotonicity of $\widetilde{V}^j_{sig}(\theta_i;k)$ comes from the fact that each voter's private preference also serves as a piece of private information about how she will vote for the revised proposal. The one with ideal point closer to the status quo believes that a revised proposal is more likely to be rejected (given the proposal constant), therefore the deviation option becomes more attractive for the voter.

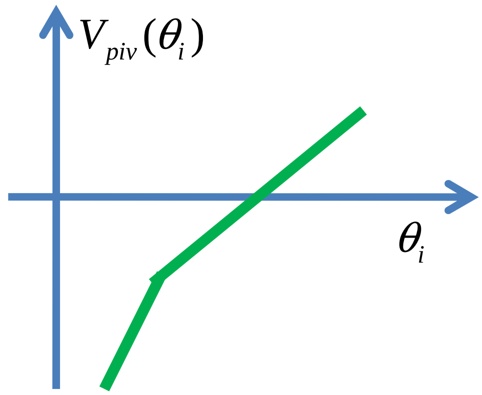

Figure 1: Pivot Effect

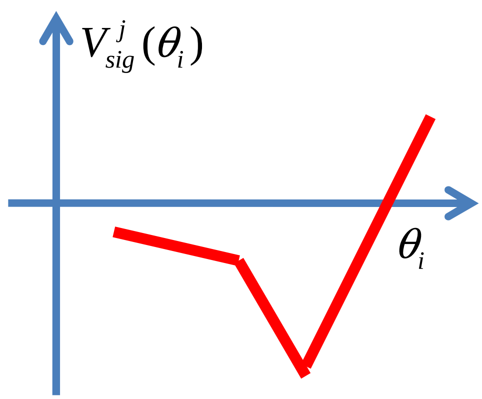

Figure 2: Signaling Effect

Since the sum of several quasi-convex (but not convex) functions may not be a quasi-convex function at all, even though we check the incentive of the lowest type $\underline{\theta}$, the incentive compatibility constraints of the other types may not be satisfied. Thus, it is technically difficult to establish the existence of a subgame informative-voting equilibrium for any initial proposal.

To complete the proof of existence, we will first show that for a sufficiently large proposal $b_1 > 0$, a subgame informative-voting equilibrium always exists. By pretending to be a supporter for an extreme reform, a voter bears a risk that there is a chance his vote is pivotal so that the first proposal gets accepted. If the setter designs the initial proposal well, she should not keep such a risk at the minimum level. As the initial proposal becomes larger, the net benefit of misleading the setter becomes lower for the voter. In this situation, the pivot effect becomes dominant, and the incentive in voting is primarily driven by the pivot effect rather than the signaling effect. In other words, the shape of the payoff gain is mainly driven by the shape of $\widetilde{V}_{piv}(\theta_i; k, b_1)$, which is strictly increasing in θ_i, providing the initial proposal b_1 is sufficiently high. Hence, although the payoff-gain function may not be monotonic, the incentive compatibility constraints of voters are automatically satisfied.

5. Main Result

As a preparation, we first provide the following lemma.

Lemma 7. *Let* $\inf \widehat{K}(b_1)$ *denote the infimum of the set of all indifferent types,* $\widehat{K}(b_1)$*, which is defined in Lemma 5. Then we have* $\lim_{b_1 \to +\infty} \inf \widehat{K}(b_1) = +\infty$.

(See Appendix for the proof.)

The lemma suggests that, as the setter increases the initial proposal, the benefit of casting a positive vote decreases, hence it will be more likely for a voter to cast a negative vote. By using the lemma, we show the following proposition.

Proposition 1. $\exists M > 0$, *such that any proposal* $b_1^0 > M$, *induces a subgame informative-voting equilibrium with the cut-point* $k^*(b_1^0) \in (0, +\infty)$. *The setter's expected payoff in any of these subgame equilibria is strictly higher than that in the case without the initial period (which is equivalent to* $k^* = +\infty$*).*

(See Appendix for the proof.)

The proposition suggests that, as the setter proposes a policy which is far away from the status quo, no voter has a profitable deviation from the cut-point strategy. As a result, a subgame informative-voting equilibrium can be induced. The second part of the proposition suggests that a sufficiently high proposal makes the setter strictly better off than in the case with only one chance of making the proposal. If the sufficiently high proposal is approved, then this is obviously better for the setter than in the case when she has only one chance to make the proposal. If it is not approved, she learns something about the preferences of the voters and can choose a better-informed proposal in the second round. And the worst case scenario is that she could propose exactly what she would have proposed if she never had the chance of making multiple proposals.

Given the above proposition, we can always construct an informative-voting equilibrium as follows: given a $b_1^0 > M$, for $b_1 = b_1^0$, the subgame-equilibrium cut-point is $k^*(b_1^0) \in (0, +\infty)$; otherwise, we have the subgame-equilibrium with $k^*(\cdot) = +\infty$. Given the voters' best response to the initial proposal, according to Proposition 1, the optimal initial proposal $b_1^* = b_1^0$.

By our construction, this informative-voting equilibrium gives the setter a strictly higher payoff than in the case without the initial period of learning.[10]

The next question is: how can we characterize the informative-voting equilibrium that gives the setter highest payoff among all the informative-voting equilibria? Specifically, how can we obtain the setter's equilibrium payoff, and the voter's cut-point on the equilibrium path?

To answer the questions, we treat the setter's problem as a mechanism design problem. Specifically she chooses (b_1, k) such that the incentive compatibility constraints of the voters are satisfied and $k > 0$ in order to maximize her own expected payoff. We define the set of (b_1, k) such that the incentive compatibility constraints of the voters are satisfied, Γ as follows.

$\Gamma = \{(b_1, k) : \frac{V_{diff}(k;k,b_1)}{F^{N-q}(k)(1-F(k))^{q-1}} = 0; V_{diff}(\theta_i; k, b_1) \geq 0 \text{ for } \theta_i \geq k; V_{diff}(\theta_i; k, b_1) \leq 0 \text{ for } \theta_i \leq k; k \in (0, +\infty); b_1 > 0\}$.[11]

It can be verified that: for any $b_1 > 0$, $k^*(b_1)$ is a subgame informative-voting equilibrium cut-point, if and only if $(b_1, k^*(b_1)) \in \Gamma$. By Proposition 1, we know $\exists\, b_1^0 \in (0, +\infty)$ and $k^0 \in (0, +\infty)$ such that $(b_1^0, k^0) \in \Gamma$, so that $\Gamma \neq \emptyset$. For (b_1, k), the setter's expected payoff is denoted as $E(U_A(b_1, k))$ given her best response in the second period $\beta(r; k)$ (which is defined in equation (5)), and voters' equilibrium voting strategies in the last stage $V^2(\theta_i, b_2)$. We will show that $\exists\, b_1^* \in (0, +\infty)$ and $\widehat{k}^* \in (0, +\infty)$ such that $(b_1^*, \widehat{k}^*) \in$

[10] In any informative-voting equilibrium, the setter's welfare is always *weakly* higher than in the case without additional chances of revising the proposals. For any equilibrium, as long as the maximizer is unique when the setter calculates the optimal b_1^* within this equilibrium, the setter has a strictly higher payoff than in the case with only one chance of making the proposal.

[11] Notice that voter's payoff gain $V_{diff}(k; k, b_1)$ already incorperates the setter's best response in the second period $\beta(r; k)$, and the voters' equilibrium voting strategies in the last stage $V^2(\theta_i, b_2)$.

$\arg\max_{(b_1,k)\in\Gamma} E(U_A(b_1,k))$. Given b_1^* and $\widehat{k}^*$, the corresponding equilibrium voting strategies can be simply recovered,[12] and this equilibrium gives the setter the highest payoff among all of the cut-point equilibria. We summarize the results in the following proposition.

Proposition 2. *(1) There exists an informative-voting equilibrium, which gives the setter a strictly higher payoff than in the case without the initial period of learning.*

(2) There exists an informative-voting equilibrium with $b_1^* \in (0, +\infty)$ *and* $k^*(b_1^*) \in (0, +\infty)$*, which gives the setter the highest payoff among all the informative-voting equilibria.*[13]

(See Appendix for the proof.)

The proposition suggests that, even though some initial proposal b_1 may not enable the setter to learn any information from voters upon the failure of the first try, the agenda-setting power allows her to design an optimal initial proposal that maximizes the value of learning. A sophisticated design of the initial proposal provides an appropriate risk for the voters and prevents them from deviating from the cut-point equilibrium, hence generating credible information as an efficient screening technology.

[12] For example, for all $b_1 \neq b_1^*$, voters reject the proposals with probability 1; otherwise they vote according to the cut-point $k^*(b_1^*) = \widehat{k}^*$.

[13] Under the general utility functions, we need to assume that there exists $\lambda > 0$ such that $v(x)^2 \leq \lambda C(x)$.

6. Discussion

In this paper, we develop a model of sequential and collective bargaining that incorporates private information about preferences. We focus on a particular form of informative equilibrium, where the voters use a cut-point strategy in the initial voting. We illustrate that the revelation of a voter' preference in the initial voting depends on three concerns: (1) his vote may be pivotal, (2) his vote provides a signal that sways the future proposal toward his ideal, and (3) his vote provides a signal to induce a more greedy proposal, which is more likely to fail. The third concern (i.e., the saboteur incentive) could dilute the incentive of preference disclosure, and reflects a technical difficulty of establishing the existence of an informative cut-point equilibrium in this game. Specifically, the payoff gain of each voter in the initial period may not be a monotonic function of his type. Although the saboteur incentive may conflict with the other two, especially for the voter who likes the status quo the best, we show that the agenda-setting power makes the saboteur incentive dominated. Hence, an informative equilibrium exists, where the revised proposal is monotonic with respect to the vote totals. The model illustrates the effect of the agenda-setting power on extracting information in a dynamic and collective-decision environment.

There are several possible extensions, which we do not make in our equilibrium analysis. First of all, we only consider the equilibrium where the voters cast a positive vote in the initial stage if and only if his ideal point is above a threshold. It will be interesting to see whether the following equilibria exist: (a) a non-monotonic equilibrium where a voter casts a positive vote when his ideal point is either extremely high or extremely low, and (b) an equilibrium where a voter casts a positive vote when his ideal point is below a threshold. Based on the techniques developed in this paper, at least we know a necessary condition of the second type of monotone equilibrium. If that equilibrium exists, the more negative votes will induce a less compromising proposal.

Another possible extension is to consider other communication protocols. In our game, the initial proposal serves as a costly screening technology. We may also consider cheap-talk communication as an alternative setup. For example, Chen and Eraslan (2013, 2014) and Chen (2014) study the cheap-talk communication before one-shot collective bargaining. From the perspective of the institutional design, which (costly or costless) communication protocol (among all possible ones) maximizes the setter's or the voters' welfare is an interesting question. In addition, in our model, the agenda setter does not have private information about her preference. It is also interesting to explore the situation when she has private information and her initial proposal as a signal may coordinate voting and communication to her own favor. Furthermore, in our model, we assume that voters' preferences are independent. We suspect that allowing certain forms of interdependence may create a new common knowledge that dilutes the saboteur incentive of voters and creates a technical convenience.

Appendix

Preparation

We first present generalized utility functions, and then show all the results based on these generalized assumptions.

Each voter's utility over policy x is[14] $u_v(x;\theta_i) = \theta_i v(x) - C(x)$. Setter's utility over policy x is $u_A(x) = \sigma v(x)$, where $\sigma > 0$. Without loss of generality, we normalize σ to 1. We use the following notations: $h(x) \triangleq \frac{C'(x)}{v'(x)}, \psi(x) \triangleq \frac{C(x)}{v(x)}$. We further assume:

Assumption 2. *(2.1)* $v(\cdot) \in C^2([0,+\infty)), C(\cdot) \in C^2([0,+\infty))$;

(2.2) $\forall x > 0, C'(x) > 0, C''(x) \geq 0, v'(x) > 0, v''(x) \leq 0, C''(x) > v''(x)$;

(2.3) $v(0) = C(0) = 0$;

(2.4) $\lim\limits_{x\to 0^+} \frac{C'(x)}{v'(x)} = 0, \lim\limits_{b\to +\infty} \frac{C(x)}{v(x)} = +\infty, \lim\limits_{x\to 0^+} v'(x) > 0$*; and*

(2.5) $\forall x > 0, (\frac{v}{v'})' + \frac{v}{v'}\frac{\psi''}{\psi'} > 0$*; in other words,* $h(x) - \psi(x) = \frac{v}{v'}\psi'$ *is strictly increasing.*

We characterize some properties of those functions in Lemma *11* and provide examples in the Supplementary Appendix.

Proof *of* **Lemma 1**

(1) When $\underline{\theta} = 0$, the result is obvious.

When $\underline{\theta} < 0$, suppose there is a subgame-equilibrium cut-point $k^*(b_1) \in (\underline{\theta}, 0]$ for some $b_1 > 0$. If there are at least q ideal points above $k^*(b_1)$, the

[14] More generally we can have $u_i(x;\theta_i) = \theta_i v(x) - C(x) + G(\theta_i)$, where $G(\cdot)$ can be any function. The functional form of $G(\cdot)$ does not change the results. Thus without loss of generality, we assume $G(\theta_i) \equiv 0$.

initial proposal gets accepted; otherwise, the "pivotal" ideal point is below $k^*(b_1) \leq 0$, and the policy proposal other than the status quo will always be rejected. From the perspective of voters, they simultaneously make a collective choice between b_1 and the status quo $s = 0$. Conditional on the pivotality, each voter's payoff gain is $\binom{N-1}{q-1} F(k^*(b_1))^{N-q}(1 - F(k^*(b_1)))^{q-1}[u_v(b_1; \theta_i) - u_v(0; \theta_i)]$. Thus the equilibrium cut-point should be $k^*(b_1) = \psi(b_1) > 0$. It is a contradiction with the assumption that $k^*(b_1) \in (\underline{\theta}, 0]$. As a result, we must have $k^*(b_1) > 0, \forall\, b_1 > 0$.

(2) When $b_1 = 0$, suppose there is subgame equilibrium with $k^*(b_1) \in [\underline{\theta}, 0]$. If there are at least q ideal points above $k^*(b_1)$, the initial proposal $b_1 = 0$ gets accepted; otherwise, the "pivotal" ideal point is below $k^*(b_1) \leq 0$, and the policy proposal other than the status quo will always be rejected. As a result, the status quo 0 will be implemented with probability 1, and the setter always gets 0. In the static game when she has one chance to make the proposal (which is equivalent to the case when $k^*(\cdot) = +\infty$), the setter always has an option to propose 0 and get 0. As a result, in the static game, her equilibrium expected payoff is weakly higher than 0.

The Revised Proposal

In the voting stage of the second period, each voter votes for the revised proposal against the status quo if and only if $\theta_i \geq \psi(b_2)$. Hence, with $r \geq N - q + 1$ negative votes in the first period, the setter's expected utility in the second period (under proposal b) is $[1 - \Omega(\psi(b)|r; k)]u_A(b) = [1 - \widetilde{F}_{r,N+1-q}(\psi(b); k)]u_A(b)$. Hence, $\beta(r; k) \triangleq \arg\max_{b \in [0,+\infty)} [1 - \Omega(\psi(b)|r; k)]u_A(b)$.

Lemma 8. *(Full Characterization of the Second-Period Proposal) For* $k \in (0, +\infty)$*, the second-period proposal* $\beta(r;k)$ *(with* $r \geq N-q+1$*) is uniquely and well determined by*

$$\frac{1-\widetilde{F}_{r,N+1-q}(\psi(b);k)}{\widetilde{f}_{r,N+1-q}(\psi(b);k)} = \frac{u_A(b)}{u'_A(b)}\psi'(b), \tag{A1}$$

and $\beta(r;k) \in (0, \psi^{-1}(k))$*. Furthermore,* $\lim_{k\to+\infty}\beta(r;k)$ *exists and is finite, and is equal to* $\arg\max_b [1-\widetilde{F}_{r,N+1-q}(\psi(b);+\infty)]u_A(b)$*. Furthermore,* $\beta(r;k)$ *is strictly decreasing in the total negative votes,* r*, and* $\beta(r;k)$ *is continuously differentiable and strictly increasing in* k*.*

Proof *of* **Lemma 8**

For convenience, we use Eu_A to represent the setter's expected utility at the beginning of the second period.

(1) We first rule out the corner solution.

$\forall b \in (0, \psi^{-1}(k))$, we have $0 < \psi(b) < k$, thus $Eu_A \mid_b > 0$, which implies that $\forall b \in [\psi^{-1}(k), +\infty) \cup \{0\}$ (which gives the setter non-positive expected payoffs) is strictly dominated by $b \in (0, \psi^{-1}(k))$.

(2) By Lemma 13 $\frac{1-\widetilde{F}_{r,N-q+1}(\psi(b))}{\widetilde{f}_{r,N-q+1}(\psi(b))}$ is strictly decreasing in b. By Lemma 11, $\frac{u_A(b)}{u'_A(b)}\psi'(b)$ is strictly increasing in b. So $\frac{1-\widetilde{F}_{r,N-q+1}(\psi(b))}{\widetilde{f}_{r,N-q+1}(\psi(b))} - \frac{u_A(b)}{u'_A(b)}\psi'(b)$ is strictly decreasing in b.

$$\begin{aligned}\frac{dEu_A}{db} &= [1-\widetilde{F}_{r,N-q+1}(\psi(b))]u'_A(b) - \widetilde{f}_{r,N-q+1}(\psi(b))\psi'(b)u_A(b) \\ &= u'_A(b)\widetilde{f}_{r,N-q+1}(\psi(b))[\frac{1-\widetilde{F}_{r,N-q+1}(\psi(b))}{\widetilde{f}_{r,N-q+1}(\psi(b))} - \frac{u_A(b)}{u'_A(b)}\psi'(b)]\end{aligned}$$

Hence, we know that $Eu_A = [1-\widetilde{F}_{r,N-q+1}(\psi(b))]u_A(b)$ is single peaked.

$\lim_{b\to0^+}[\frac{1-\widetilde{F}_{r,N-q+1}(\psi(b))}{\widetilde{f}_{r,N-q+1}(\psi(b))} - \frac{u_A(b)}{u'_A(b)}\psi'(b)] > 0$, $\lim_{b\to\psi^{-1}(k)}[\frac{1-\widetilde{F}_{r,N-q+1}(\psi(b))}{\widetilde{f}_{r,N-q+1}(\psi(b))} - \frac{u_A(b)}{u'_A(b)}\psi'(b)] < 0$. As a result, $\beta(r;k)$ is uniquely and well determined by the F.O.C.

By taking $k \to +\infty$ in F.O.C., we get the first-order condition of $\max_b [1 - \widetilde{F}_{r,N+1-q}(\psi(b);+\infty)]u_A(b)$. As a result, $\lim_{k\to+\infty} \beta(r;k) = \arg\max_b [1 - \widetilde{F}_{r,N+1-q}(\psi(b);+\infty)]u_A(b)$ exists and is finite.

(3) Define $G(b,k,r) = \frac{1-\widetilde{F}_{r,N+1-q}(\psi(b);k)}{\widetilde{f}_{r,N+1-q}(\psi(b);k)} - \frac{u_A(b)}{u'_A(b)}\psi'(b)$ for $b \in (0,+\infty)$. Thus $\beta(r;k)$ is uniquely and well defined by $G(b,k,r) = 0$. Suppose $r_1 > r_2$, we want to show that $\beta(r_1;k) < \beta(r_2;k)$. If it is not true, then we have $\beta(r_1;k) \geq \beta(r_2;k)$. Therefore we have $G(\beta(r_2;k),k,r_2) \geq G(\beta(r_1;k),k,r_2)$, by Lemma 13 and $G(\beta(r_1;k),k,r_2) > G(\beta(r_1;k),k,r_1)$, by Lemma 11. The two inequalities imply that $G(\beta(r_2;k),k,r_2) > G(\beta(r_1;k),k,r_1) = 0$. It is a contradiction. As a result, we must have $\beta(r_1;k) < \beta(r_2;k)$ so that $\beta(r;k)$ is strictly decreasing in r.

Similarly we can also show that $\beta(r;k)$ is strictly increasing in k.

(4) Because $\frac{1-\widetilde{F}_{r,N+1-q}(\psi(b);k)}{\widetilde{f}_{r,N+1-q}(\psi(b);k)}$ is continuously differentiable in b and k, $G(b,k,r)$ is continuously differentiable in b and k. We know that $\frac{\partial G(b,k,r)}{\partial b} < 0$, therefore, by Implicit Function Theorem, we know that $\beta(r;k)$ is continuously differentiable at any $k > 0$. Q.E.D.

Voting Incentives

Lemma 9. *If $b_1 = 0$ induces a subgame-equilibrium cut-point $k^*(0) \in (0,+\infty)$, then $b_1 = \psi^{-1}(k^*(0)) > 0$ induces a subgame equilibrium with the same cut-point $k^*(b_1) = k^*(0)$ and makes the agenda setter strictly better off than in the case with $b_1 = 0$.*

Proof *of* **Lemma 9**

Suppose $k^* \in (0,+\infty)$ is a subgame-equilibrium cut-point induced by $b_1 = 0$, we must have $V_{diff}(\theta_i = k^*; k^*, b_1 = 0) = 0$. Because $v(\psi^{-1}(k^*))k^* -$

$C(\psi^{-1}(k^*)) = 0 = v(0)k^* - C(0)$, we have $V_{diff}(\theta_i = k^*; k^*, b_1 = \psi^{-1}(k^*)) = 0$, so that $\theta_i = k^*$ is indifferent between voting for $b_1 = \psi^{-1}(k^*)$ and against it. In the following, we check the incentives of the other types.

The change of the initial proposal to $\psi^{-1}(k^*)$ does not affect the signaling incentives, i.e., the function $V_{sig}(\theta_i, k^*)$.

For $\theta_i > k^*$, $v(\psi^{-1}(k^*))\theta_i - C(\psi^{-1}(k^*)) > v(\psi^{-1}(k^*))k^* - C(\psi^{-1}(k^*)) = 0$, therefore $V_{diff}(\theta_i; k^*, b_1 = \psi^{-1}(k^*)) > V_{diff}(\theta_i; k^*, b_1 = 0)$.

For $\theta_i < k^*$, $v(\psi^{-1}(k^*))\theta_i - C(\psi^{-1}(k^*)) < v(\psi^{-1}(k^*))k^* - C(\psi^{-1}(k^*)) = 0$, therefore $V_{diff}(\theta_i; k^*, b_1 = \psi^{-1}(k^*)) < V_{diff}(\theta_i; k^*, b_1 = 0)$.

As a result, under $b_1 = \psi^{-1}(k^*)$, no voters have an incentive to deviate from the cut-point strategy k^*. With the same probability, the initial proposal will be accepted. However once it is accepted, the setter becomes strictly better off. Hence, $b_1 = \psi^{-1}(k^*)$ provides a strictly higher expected payoff for the setter. Q.E.D.

A generalized version of Equation (7) and Equation (10) is

$$V(\theta_i; b_2, N-q, k) = \begin{cases} u_v(b_2; \theta_i) & \text{if } \theta_i \geq \psi(b_2) \\ [1 - \widetilde{F}_{N-q,N-q}(\psi(b_2); k)]u_v(b_2; \theta_i) & \text{if } \theta_i < \psi(b_2) \end{cases}, \tag{A2}$$

$$V(\theta_i; b_2, N-1-j, k) = \begin{cases} [1 - \widetilde{F}_{N-1-j,N+1-q}(\psi(b_2); k)]u_v(b_2; \theta_i) & \text{if } \theta_i \geq \psi(b_2) \\ [1 - \widetilde{F}_{N-1-j,N-q}(\psi(b_2); k)]u_v(b_2; \theta_i) & \text{if } \theta_i < \psi(b_2) \end{cases}. \tag{A3}$$

Let's define

$$\widetilde{\Pi}_{r,t}(x; k) = [1 - \widetilde{F}_{r,t}(\psi(x); k)]v(x), \tag{A4}$$

$$\widetilde{Y}_{r,t}(x; k) = [1 - \widetilde{F}_{r,t}(\psi(x); k)]C(x). \tag{A5}$$

Functions $\widetilde{V}_{piv}$ and $\widetilde{V}_{sig}^{j}$ can be rewritten as

$$\widetilde{V}_{piv}(\theta_i;k,b_1)=\begin{cases}[v(b_1)-v(b_2)]\theta_i-[C(b_1)-C(b_2)] & \text{if} \quad \theta_i\geq\psi(b_2)\\ [v(b_1)-\widetilde{\Pi}_{N-q,N-q}(b_2;k)]\theta_i-[C(b_1)-\widetilde{Y}_{N-q,N-q}(b_2;k)] & \text{if} \quad \theta_i<\psi(b_2)\end{cases}, \tag{A6}$$

where

$$b_2=\beta(N-q+1;k); \tag{A7}$$

$$\widetilde{V}_{sig}^{j}(\theta;k)=\begin{cases}W_{1j}\theta-E_{1j} & \text{if } \theta_i\geq\psi(\beta(N-j-1;k))\\ W_{2j}\theta-E_{2j} & \text{if } \psi(\beta(N-j;k))\leq\theta_i<\psi(\beta(N-j-1;k))\\ W_{3j}\theta-E_{3j} & \text{if } \theta_i<\psi(\beta(N-j;k))\end{cases}, \tag{A8}$$

where

$$W_{1j}=\widetilde{\Pi}_{N-j-1,N-q+1}(\beta(N-j-1;k);k)-\widetilde{\Pi}_{N-j-1,N-q+1}(\beta(N-j;k);k), \tag{A9}$$

$$W_{2j}=\widetilde{\Pi}_{N-j-1,N-q}(\beta(N-j-1;k))-\widetilde{\Pi}_{N-j-1,N-q+1}(\beta(N-j;k);k), \tag{A10}$$

$$W_{3j}=\widetilde{\Pi}_{N-j-1,N-q}(\beta(N-j-1;k);k)-\widetilde{\Pi}_{N-j-1,N-q}(\beta(N-j;k);k), \tag{A11}$$

$$E_{1j}=\widetilde{Y}_{N-j-1,N-q+1}(\beta(N-j-1;k);k)-\widetilde{Y}_{N-j-1,N-q+1}(\beta(N-j;k)), \tag{A12}$$

$$E_{2j}=\widetilde{Y}_{N-j-1,N-q}(\beta(N-j-1;k);k)-\widetilde{Y}_{N-j-1,N-q+1}(\beta(N-j;k);k), \tag{A13}$$

$$E_{3j}=\widetilde{Y}_{N-j-1,N-q}(\beta(N-j-1;k);k)-\widetilde{Y}_{N-j-1,N-q}(\beta(N-j;k);k). \tag{A14}$$

Proof *of* **Lemma 5**

It can be verified that

$$\frac{V_{diff}(\theta_i = k; k, b_1)}{F^{N-q}(k)(1-F(k))^{q-1}} = \begin{cases} \binom{N-1}{q-1}[(v(b_1) - v(b_2))k - (C(b_1) - C(b_2))] \\ +\sum_{j=0}^{q-2}\binom{N-1}{j}[\frac{F(k)}{1-F(k)}]^{q-1-j}[2W_{1j}k - E_{1j}] \end{cases}, \tag{A15}$$

and

$$\frac{V_{diff}(\theta_i = k; k, b_1)}{F^{N-q}(k)(1-F(k))^{q-1}k} = \begin{cases} \binom{N-1}{q-1}[(v(b_1) - v(b_2)) - \frac{C(b_1)-C(b_2)}{k}] \\ +\sum_{j=0}^{q-2}\binom{N-1}{j}[\frac{F(k)}{1-F(k)}]^{q-1-j}[2W_{1j} - \frac{E_{1j}}{k}] \end{cases}, \tag{A16}$$

where $b_2 = \beta(N - q + 1; k)$.

(1) As $k \to +\infty$, $\beta(N-q+1, k)$ approaches a finite number according to Lemma 8, therefore $\binom{N-1}{q-1}[(v(b_1) - v(b_2)) - \frac{C(b_1)-C(b_2)}{k}]$ converges to a finite number. Similarly $\lim_{k\to+\infty} E_{1i}(k)$ exists and is finite, so that $\lim_{k\to+\infty} \frac{E_{1i}(k)}{k} = 0$. $\lim_{k\to+\infty} \widetilde{\Pi}_{N-j-1,N-q+1}(\beta(N-j-1;k);k)$ is the maximum payoff the setter gets in the static game with $(N-j-1)$ voters whose ideal points are i.i.d. drawn from $F(\cdot)$ and with the q rule. Since $\lim_{k\to+\infty}\beta(N-j-1;k) > \lim_{k\to+\infty}\beta(N-j;k)$, and $\lim_{k\to+\infty}\beta(N-j-1;k)$ is the unique maximizer, $\lim_{k\to+\infty}\widetilde{\Pi}_{N-j-1,N-q+1}(\beta(N-j-1;k);k) > \lim_{k\to+\infty}\widetilde{\Pi}_{N-j-1,N-q+1}(\beta(N-j;k);k)$. Hence, $\lim_{k\to+\infty} W_{1j} > 0$ and $\lim_{k\to+\infty} \frac{V_{diff}(\theta_i=k;k,b_1)}{F^{N-q}(k)(1-F(k))^{q-1}k} = +\infty$.

(2) According to Lemma 8, $0 < \beta(\cdot;k) < \psi^{-1}(k)$, we have $\lim_{k\to 0}\beta(\cdot;k) = 0$. Thus $\lim_{k\to 0}(W_{1j}k - E_{1j}) = 0$ and $\lim_{k\to 0}\frac{V_{diff}(\theta_i=k;k,b_1)}{F^{N-q}(k)(1-F(k))^{q-1}} = -\binom{N-1}{q-1}C(b_1) < 0$ for $b_1 > 0$.

(3) Because $V_{diff}(\theta_i = k;k;b_1)$ is continuous in k, there exists a $\widehat{k} \in (0, +\infty)$ such that $V_{diff}(\theta_i = \widehat{k}; \widehat{k}, b_1) = 0$.Q.E.D.

Lemma 10. *(Non-monotonicity of the Signaling Payoff Gain) Suppose* $k \in (0, +\infty)$. W_{1j}, W_{2j}, W_{3j}, E_{3j} *are defined by Equations (A9), (A10), (A11), (A14), and* $\widetilde{V}_{piv}(\theta_i; k)$ *is characterized by Equation (A6).*

(1) For $j = 0, 1, ...(q-2), W_{1j} > 0$, $W_{2j} < W_{3j} < 0$, *when* $q < N$; $W_{1j} > 0$, $W_{2j} < 0$, $W_{3j} = E_{3j} = 0$, *when* $q = N$; *and*

(2) $\widetilde{V}_{piv}(\theta_i; k)$ *is strictly increasing in* θ_i, *provided* $b_1 > \beta(N - q + 1; k = +\infty)$

Proof *of* **Lemma 10**

Because (2) is obvious from the expression of $\widetilde{V}_{piv}(\theta_i; k)$, we only need to show (1).

(a) Because $\beta(N-1-j;k) = \arg\max_x \widetilde{\Pi}_{N-1-j,N+1-q}(x;k)$, it is obvious that $W_{1j} = \widetilde{\Pi}_{N-1-j,N+1-q}(\beta(N-1-j;k);k) - \widetilde{\Pi}_{N-1-j,N+1-q}(\beta(N-j;k);k) > 0$.

(b) $\because$ $1 - \widetilde{F}_{r,t}(x) = \sum_{i=0}^{t-1} \binom{r}{j} \widetilde{F}^j (1-\widetilde{F})^{r-j}, \widetilde{\Pi}_{r,t}(b_2;k) = [1 - \widetilde{F}_{r,t}(\psi(x);k)]v(x)$ $\therefore$ $\widetilde{\Pi}_{r,t+1}(b_2;k) > \widetilde{\Pi}_{r,t}(b_2;k)$ for any b_2.

$\because$ $\widetilde{\Pi}_{N-1-j,N+1-q}(b_2;k) > \widetilde{\Pi}_{N-1-j,N-q}(b_2;k)$ $\therefore$ $\widetilde{\Pi}_{N-1-j,N+1-q}(\beta(N-j;k);k) > \widetilde{\Pi}_{N-1-j,N-q}(\beta(N-j;k);k)$, i.e., $W_{2j} < W_{3j}$.

(c) Suppose $N > q$. According to the similar logic as in the proof of Lemma 8, $\widetilde{\Pi}_{N-1-j,N-q}(x;k)$ is a single-peaked function in x. In the following, we will show that $\arg\max_x \widetilde{\Pi}_{N-1-j,N-q}(x;k) < \beta(N-j;k)$. Because $\beta(N-1-j;k) > \beta(N-j;k)$, we then have $\widetilde{\Pi}_{N-1-j,N-q}(\beta(N-1-j;k);k) < \widetilde{\Pi}_{N-1-j,N-q}(\beta(N-j;k);k)$, so that $W_{3j} < 0$.

Recall that $\beta(N-j;k) = \arg\max_x \widetilde{\Pi}_{N-j,N-q+1}(x;k)$, and it is determined by $\frac{1-\widetilde{F}_{N-j,N+1-q}(\psi(b);k)}{\widetilde{f}_{N-j,N+1-q}(\psi(b);k)} = \frac{u_A(b)}{u'_A(b)}\psi'(b)$. $\arg\max_x \widetilde{\Pi}_{N-1-j,N-q}(x;k)$ is determined by $\frac{1-\widetilde{F}_{N-j-1,N-q}(\psi(b);k)}{\widetilde{f}_{N-j-1,N+1-q}(\psi(b);k)} = \frac{u_A(b)}{u'_A(b)}\psi'(b)$. According to Lemma 12 in the Supplementary Appendix, we know $\frac{1-\widetilde{F}_{N-j,N+1-q}(\psi(b);k)}{\widetilde{f}_{N-j,N+1-q}(\psi(b);k)} > \frac{1-\widetilde{F}_{N-j-1,N-q}(\psi(b);k)}{\widetilde{f}_{N-j-1,N-q}(\psi(b);k)}$. Hence, $\beta(N-j;k) > \arg\max_x \widetilde{\Pi}_{N-1-j,N-q}(x;k)$.

As a result, $W_{3j} = \widetilde{\Pi}_{N-1-j,N-q}(\beta(N-1-j;k);k) < \widetilde{\Pi}_{N-1-j,N-q}(\beta(N-j;k);k) < 0$.

(d) Suppose $N = q$, $E_{3j} = \widetilde{Y}_{N-j-1,0}(\beta(N-j-1;k);k) - \widetilde{Y}_{N-j-1,0}(\beta(N-j;k);k) = 0$ since we defined $\widetilde{F}_{r,0}(x) = 1$. Q.E.D.

Proof *of* Lemma 7

We show it by contradiction. Suppose $\underline{\lim}_{b_1\to+\infty} \inf \widehat{K}(b_1)$ is a finite number k_0. Then there exists $\hat{k}(t) \in \widehat{K}(t)$, where $t = 1, 2, 3, ...$, such that $\lim_{t\to+\infty} \hat{k}(t) = k_0$.

According to Equation (A15), we have

$$0 = \left\{ \begin{array}{c} \binom{N-1}{q-1}[(v(t) - v(b_2))k - (C(t) - C(b_2))] \\ +\sum_{j=0}^{q-2}\binom{N-1}{j}[\frac{F(\hat{k}(t))}{1-F(\hat{k}(t))}]^{q-1-j}[2W_{1j}(\hat{k}(t))\hat{k}(t) - E_{1j}(\hat{k}(t))] \end{array} \right. . \tag{A17}$$

As t goes to infinity, the first term of the right-hand side goes to negative infinity, whereas the second term of the right-hand side goes to a finite number. As a result, the above equation cannot hold. It is a contradiction. As a result, we must have $\underline{\lim}_{b_1\to+\infty} \inf \widehat{K}(b_1) = +\infty$. Q.E.D.

Proof *of* Proposition 1

For any $b_1 > 0$, pick an element $\hat{k}(b_1)$ from the set $\widehat{K}(b_1)$. By Lemma 7, we have $\lim_{b_1\to+\infty} \hat{k}(b_1) = +\infty$.

We construct a function:

$$M(\theta_i;\hat{k}(b_1)) \triangleq V_{piv}(\theta_i;\hat{k}(b_1))+\sum_{j=0}^{q-2}P_{2j}\max\{2W_{1j}\theta_i-E_{1i},2W_{3i}\underline{\theta}-E_{3i}\}. \quad \text{(A18)}$$

W_{1j}, E_{1i}, W_{3i} and E_{3i} are evaluated at $k=\hat{k}(b_1)$.

$M^j(\theta_i) \triangleq \max\{2W_{1j}\theta_i - E_{1i}, 2W_{3i}\underline{\theta} - E_{3i}\}$ is drawn in Figure *3*.

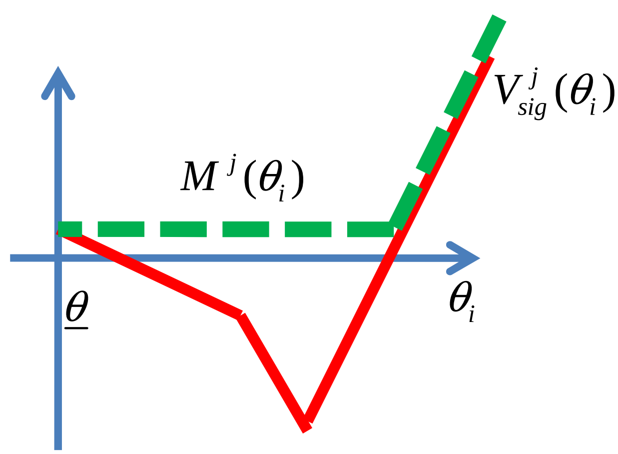

Figure 3: The Constructed Function

As b_1 becomes sufficiently large, $\hat{k}(b_1)$ is also sufficiently large, therefore $2W_{1j}\hat{k}(b_1) - E_{1i} > 2W_{3i}\underline{\theta} - E_{3i}$ for $j = 0, ..., q-2$. So we have

$M(\theta_i = \hat{k}(b_1)); \hat{k}(b_1)) = V_{diff}(\theta_i = \hat{k}(b_1)); \hat{k}(b_1), b_1) = 0.$

By construction, $M(\theta_i; \hat{k}(b_1))$ is an increasing function in θ_i for sufficiently large b_1. Hence,

whenever $\theta_i > \hat{k}(b_1)$, $V_{diff}(\theta; k^*(b_1), b_1) = M(\theta_i; \hat{k}(b_1)) > M(\theta_i = \hat{k}(b_1); \hat{k}(b_1)) = 0$;

whenever $\theta_i < \hat{k}(b_1)$, $V_{diff}(\theta; k^*(b_1), b_1) \leq M(\theta_i; \hat{k}(b_1)) < M(\theta_i = \hat{k}(b_1); \hat{k}(b_1)) = 0$.

As a result, $\hat{k}(b_1)$ is a subgame informative-voting equilibrium cut-point for sufficiently large b_1. Under this proposal b_1 and the cut-point $\hat{k}$, we have $v(b_1) > v(b')$, where b' is the optimal proposal the setters makes without learning. Furthermore, for $j \leq q-1$, by the definition of $\beta(N-j; k)$, we have

$[1 - \widetilde{F}_{N-j,N-q+1}(\psi(\beta(N-j;\hat{k})); \hat{k})]v(\beta(N-j;\hat{k})) \geq [1 - \widetilde{F}_{N-j,N-q+1}(\psi(b'); \hat{k})]v(b')$,

and at least one of the inequalities strictly holds.

Thus, the setter's expected welfare

$\sum_{j=q}^{N} \binom{N}{j} F(\hat{k})^{N-j}[1-F(\hat{k})]^j v(b_1) + \sum_{j=0}^{q-1} \binom{N}{j} F(\hat{k})^{N-j}[1-F(\hat{k})]^j [1 - \widetilde{F}_{N-j,N-q+1}(\psi(\beta(N-j;\hat{k})); \hat{k})]v(\beta(N-j;\hat{k}))$

$> \sum_{j=q}^{N} \binom{N}{j} F(\hat{k})^{N-j}[1-F(\hat{k})]^j v(b') + \sum_{j=0}^{q-1} \binom{N}{j} F(\hat{k})^{N-j}[1-F(\hat{k})]^j [1 - \widetilde{F}_{N-j,N-q+1}(\psi(b'); \hat{k})]v(b')$

$= [1 - F_{N,N-q+1}(\psi(b'))]v(b')$,

which is her expected payoff without the initial period of learning. Q.E.D.

Proof *of* **Proposition 2**

(0) First of all, we can re-write Γ as follows.

$\Gamma = \{(b_1, k) : \frac{V_{diff}(k;k,b_1)}{F^{N-q}(k)(1-F(k))^{q-1}} = 0; V_{diff}(\psi(\beta(r;k)); k, b_1) \leq 0$ for $r = N-q+1, ..., N; V_{diff}(\underline{\theta}; k, b_1) \leq 0; k > 0; b_1 > 0\}$.

We now show that: if $\arg\max_{(b_1,k)\in\Gamma} E(U_A(b_1, k)) \neq \emptyset$, then $\arg\max_{(b_1,k)\in\widetilde{\Gamma}} E(U_A(b_1, k)) = \arg\max_{(b_1,k)\in\Gamma} E(U_A(b_1, k))$, where

$\widetilde{\Gamma} = \{(b_1, k) : \frac{V_{diff}(k;k,b_1)}{F^{N-q}(k)(1-F(k))^{q-1}} = 0; V_{diff}(\psi(\beta(r;k)); k, b_1) \leq 0$ for $r = N-q+1, ..., N; V_{diff}(\underline{\theta}; k, b_1) \leq 0; k \geq 0; b_1 \geq 0\}$.[15]

The only possible elements that belong to $\widetilde{\Gamma}$ but not Γ are: $(b_1, 0)$ with $b_1 > 0$, $(0, k)$ with $k \geq 0$. According to Lemma 1, $b_1 > 0$ never induces a subgame informative-voting equilibrium with the cut-point $k = 0$. Hence the only possibility is $(0, k)$ with $k \geq 0$.

According to Lemma 9, if $b_1 = 0$ induces a subgame informative-voting equilibrium with the cut-point $k \in (0, +\infty)$, then $b_1 = 0$ induces a subgame informative-voting equilibrium with the cut-point $\psi^{-1}(k)$, and gives the setter a strictly higher payoff, thus $E(U_A(b_1 = 0, k)) < E(U_A(b_1 = 0, \psi^{-1}(k))) \leq \max_{(b_1,k)\in\Gamma} E(U_A(b_1, k))$, provided $k \in (0, +\infty)$. When $k = 0$, it is obvious that $E(U_A(b_1 = 0, k)) < \max_{(b_1,k)\in\Gamma} E(U_A(b_1, k))$. As a result, we must have $\arg\max_{(b_1,k)\in\widetilde{\Gamma}} E(U_A(b_1, k)) = \arg\max_{(b_1,k)\in\Gamma} E(U_A(b_1, k))$.

The agenda setter's expected utility $E(U_A(b_1, k)) = \sum_{j=q}^{N} \binom{N}{j} F(k)^{N-j}[1 - F(k)]^j v(b_1) + \sum_{j=0}^{q-1} \binom{N}{j} F(k)^{N-j}[1-F(k)]^j[1-\widetilde{F}_{j,N-q+1}(\psi(\beta(N-j)))]v(\beta(N-j))$ is continuous in (b_1, k), where $\widetilde{F}_{j,N-q+1}(\psi(\beta(N-j)))$ is short for $\widetilde{F}_{j,N-q+1}(\psi(\beta(N-j;k)); k)$ and $\beta(N-j)$ is short for $\beta(N-j; k)$.

[15] To define $\beta(r; 0)$, we use the continuous extention such that $\beta(r; 0) = \lim_{k\to 0^+} \beta(r; k) = 0$. Therefore, $\beta(r; k)$ as a function of k, is continuous on $[0, +\infty)$.

(1) We show that $\widetilde{\Gamma}$ is a closed set. We have

$\widetilde{\Gamma} = \{(b_1, k) : \frac{V_{diff}(k;k,b_1)}{F^{N-q}(k)(1-F(k))^{q-1}} = 0; V_{diff}(\psi(\beta(r;k)); k, b_1) \leq 0$ for $r = N-q+1, ..., N; V_{diff}(\underline{\theta}; k, b_1) \leq 0; k \geq 0; b_1 \geq 0\}$.

Pick any $(b_1(n), k(n)) \in \widetilde{\Gamma}$ such that $(b_1(n), k(n)) \to (b_1', k')$ as $n \to +\infty$. It can be verified that $(b_1', k') \in \widetilde{\Gamma}$. This is from the fact that $\frac{V_{diff}(k;k,b_1)}{F^{N-q}(k)(1-F(k))^{q-1}}, V_{diff}(\psi(\beta(r;k)); k, b_1)$ and $V_{diff}(\underline{\theta}; k, b_1)$ are continuous in (b_1, k), and the fact that all the inequalities used to define $\widetilde{\Gamma}$ are in the weak forms (i.e., including "="). So $\widetilde{\Gamma}$ is a closed set.

(2) In (4), We will show that $\lim_{(b_1,k)\to(\infty,\infty) \text{ and } (b_1,k)\in\widetilde{\Gamma}} E(U_A(b_1, k)) = E(U_A(\cdot, +\infty))$. Suppose this is true, then $\exists M_0 > 0$, s.t. $\forall b_1 \geq M_0, k \geq M_0$, we have $E(U_A(b_1, k)) < E(U_A(b_1^0, k^0))$, where b_1^0 and k^0 are defined in Proposition 1. It suggests that $\max_{(b_1,k)\in\widetilde{\Gamma}} E(U_A(b_1, k)) = \max_{(b_1,k)\in\widehat{\Gamma}} E(U_A(b_1, k))$, where $\widehat{\Gamma} = \widetilde{\Gamma} \cap \{(b_1, k) : b_1 \leq M_0, k \leq M_0\}$. Because $E(U_A(b_1, k))$ is continuous and $\widehat{\Gamma} = \widetilde{\Gamma} \cap \{(b_1, k) : b_1 \leq M_0, k \leq M_0\}$ is a compact set, there exists $(b_1^*, k^*) \in \widehat{\Gamma}$ such that $(b_1^*, k^*) \in \arg\max_{(b_1,k)\in\widehat{\Gamma}} E(U_A(b_1, k))$. Because $(b_1 = 0, k^*)$ is a strictly dominated strategy according to Lemma 9 and Lemma 1, we must have $b_1^* > 0$. According to the proof of Lemma 5, we know that: if $(b_1^*, k^*) \in \widetilde{\Gamma}$, and $b_1^* > 0$, we must have $k^* > 0$. As a result, there exists $(b_1^*, k^*) \in \widetilde{\Gamma}$ such that $(b_1^*, k^*) \in \arg\max_{(b_1,k)\in\widehat{\Gamma}} E(U_A(b_1, k))$, and $b_1^* > 0, k^* > 0$.

(3) As a preparation, we show $\lim_{k\to+\infty} [1 - F(k)]k = 0$.

$E(\theta_i) = \int_{\underline{\theta}}^{k} \theta_i dF(\theta_i) + \int_{k}^{+\infty} \theta_i dF(\theta_i) \geq \int_{\underline{\theta}}^{k} \theta_i dF(\theta_i) + \int_{k}^{+\infty} k dF(\theta_i) = \int_{\underline{\theta}}^{k} \theta_i dF(\theta_i) + (1 - F(k))k$. Thus, $0 \leq (1 - F(k))k \leq E(\theta_i) - \int_{\underline{\theta}}^{k} \theta_i dF(\theta_i)$.

By letting $k \to +\infty$, we have $(1 - F(k))k \to 0$.

(4) To show $\lim_{(b_1,k)\to(\infty,\infty) \text{ and } (b_1,k)\in\widetilde{\Gamma}} E(U_A(b_1, k)) = E(U_A(\cdot, +\infty))$, we only

need to show that $\lim\limits_{(b_1,k)\to(\infty,\infty) \text{ and } (b_1,k)\in\widetilde{\Gamma}} [1-F(k)]^q v(b_1) = 0$. $V_{diff}(\theta_i = k; k, b_1) = 0$ implies that $\frac{V_{diff}(\theta_i=k;k,b_1)}{v(b_1)}(1-F(k)) = 0$, i.e.,

$$0 = \left\{ \begin{array}{l} \binom{N-1}{q-1} F^{N-q}(k)(1-F(k))^q[(k-\psi(b_1)) - (v(b_2)\frac{k}{v(b_1)} - \frac{C(b_2)}{v(b_1)})] \\ +\sum\limits_{j=0}^{q-2} \binom{N-1}{j}\binom{N-1}{j} F^{N-1-j}(k)[1-F(k)]^{j+1}[2W_{1j}k - E_{1j}]\frac{1}{v(b_1)} \end{array} \right. . \tag{A19}$$

Since $\lim\limits_{k\to+\infty}[1-F(k)]k = 0$, as $(b_1,k)\to(+\infty,+\infty)$ within the set $\widetilde{\Gamma}$, we have $\lim\limits_{(b_1,k)\to(\infty,\infty) \text{ and } (b_1,k)\in\widetilde{\Gamma}} (1-F(k))^q\psi(b_1) = 0$. Thus as long as $v(b_1) \leq \lambda\psi(b_1)$, we have $\lim\limits_{(b_1,k)\to(\infty,\infty) \text{ and } (b_1,k)\in\widetilde{\Gamma}} (1-F(k))^q v(b_1) = 0$.

(5) As the last step, we use (b_1^*, k^*) we found in step (2) to construct an equilibrium. In this equilibrium, if the setter proposes b_1^*, k^* is the induced subgame informative-voting equilibrium cut-point; otherwise if the setter proposes any other policy, the voters reject it for sure. We can verify that all the incentive compatibility constraints are satisfied. Suppose there is another equilibrium with $k^{**}(b_1^{**})$ that makes the setter strictly better off, then we get $E(U_A((b_1^{**}, k^{**}))) > \max\limits_{(b_1,k)\in\widetilde{\Gamma}} E(U_A(b_1,k)) = E(U_A((b_1^*, k^*)))$. It is a contradiction. As a result, the equilibrium we construct does give the setter the highest payoff among all the informative-voting equilibrium. Q.E.D.

Supplementary Appendix

This appendix supplements *Signaling and Learning in Collective Bargaining* and will be made available on the web. Here we provide detailed proofs for some technical results. We do not cover the results that are proven directly in the paper or Appendix attached with the paper.

Example 1. *One can verify that the following functions (including quadratic utility as a special case) satisfy Assumption 2.*[1]

$v(x) = \frac{1}{\mu}x^{\alpha}, C(x) = x^{A}$

where $\alpha \in (0, 1], A \in [1, +\infty), A > \alpha, \mu > 0.$

We provide the following lemma for further technical convenience.

Lemma 11. *Under Assumption 2,*

(1) $\psi(x) \leq h(x)$, *"=" holds if and only if* $x = 0$;

(2) $\lim\limits_{x\to 0^+} \psi(x) = 0$, $\lim\limits_{x\to +\infty} \psi(x) = +\infty$, $\lim\limits_{x\to 0^+} h(x) = 0$, $\lim\limits_{x\to +\infty} h(x) = +\infty$;

(3) $\psi(x)$ *and* $h(x)$ *are strictly increasing and twice continuously differentiable;*

(4) $\theta_i v(x) - C(x)$ *is concave and thus single-peaked with ideal point* $h^{-1}(\theta_i)$ *when* $\theta_i \geq 0$; *and*

(5) $\frac{u_A(x)}{u'_A(x)}\psi'(x)$ *is strictly increasing and continuously differentiable, and* $\lim\limits_{x\to 0^+} \frac{u_A(x)}{u'_A(x)}\psi'(x) = 0$.

[1] $v(x) = \frac{1}{\mu}x^{\alpha}, C(x) = x^{A}, \psi(x) = \mu x^{A-\alpha}, h(x) = \frac{\mu A}{\alpha}x^{A-\alpha}$. We have $\frac{v}{v'} = \frac{1}{\alpha}x, \frac{\psi''}{\psi'} = \frac{(A-\alpha-1)}{x}, (\frac{v}{v'})' + \frac{v}{v'}\frac{\psi''}{\psi'} = \frac{A-\alpha}{\alpha}$.

Proof *of* **Lemma 11**

For (1) and (2), we only need to show (1).

$\forall x > 0, \exists \xi_1, \xi_2 \in [0, x], s.t. C(x) = C'(\xi_1)x \le C'(x)x, v(x) = v'(\xi_2)x \ge v'(x)x$. Therefore we get $\frac{C(x)}{v(x)} \le \frac{C'(x)}{v'(x)}$, i.e., $\psi(x) \le h(x)$.

Suppose for some $x > 0$, $\psi(x) = h(x)$. We must have $C(x) = C'(x)x, v(x) = v'(x)x$. $\because C(\cdot)$ is convex$\therefore \forall t \epsilon [0, x], C'(t) \le C'(x) = \frac{C(x)}{x}$.

$\because \int_0^x C'(t)dt = C(x) = \int_0^x \frac{C(x)}{x} dt \therefore \int_0^x [\frac{C(b)}{b} - C'(t)]dt = 0$.

$\because \frac{C(x)}{x} \ge C'(t)$ and $C'(t)$ is continuous $\therefore \frac{C(x)}{x} = C'(t), \forall t \epsilon [0, x] \therefore C''(t) = 0, \forall t \in [0, x] \therefore v''(t) < 0, \forall t \in [0, x]$.

Therefore we get $v(x) > v'(x)x$, which is a contradiction. As a result, $\psi(x) = h(x)$ if and only if $x = 0$.

(3) Assumption *2* directly implies that $\psi(x)$ and $h(x)$ are twice continuously differentiable because C and v are smooth functions.

$h'(x) = \frac{C''v' - v''C'}{(v')^2} > 0$ for $x > 0$; $\psi'(x) = \frac{C'v - v'C}{v^2} = vv'\frac{h - \psi}{v^2} > 0$ for $x > 0$.

(4) and (5) are obvious. Q.E.D.

Lemma 12. *(Basic Properties of Order Statistics) Suppose $F_{x,y}$ is the distribution function representing the y th smallest random variable among the x i.i.d. random variables with the distribution $F(\cdot)$ and the probability density function $f(\cdot), (x, y \in \mathbb{Z}^+, x \ge y)$, then we have:*

(1) $\frac{1 - F_{x,y}(\theta)}{f_{x,y}(\theta)} = \sum_{i=0}^{y-1} \frac{(y-1)!(x-y)!}{(x-i)!(i)!} (\frac{1 - F(\theta)}{F(\theta)})^{y-i-1} \frac{1 - F(\theta)}{f(\theta)}$;

(2)$\frac{1 - F_{x,y}(\theta)}{f_{x,y}(\theta)}$ *is decreasing in θ provided $F(\cdot)$ satisfies the increasing-hazard-rate property; when $y \ge 2$,* $\frac{1 - F_{x,y}(\theta)}{f_{x,y}(\theta)}$ *is strictly decreasing in θ;*

(3)$\frac{1 - F_{x,y}(\theta)}{f_{x,y}(\theta)} > \frac{1 - F_{x+1,y}(\theta)}{f_{x+1,y}(\theta)}$; *and*

(4)$\frac{1 - F_{x,y}(\theta)}{f_{x,y}(\theta)} < \frac{1 - F_{x,y+1}(\theta)}{f_{x,y+1}(\theta)}$ *and* $\frac{1 - F_{x,y}(\theta)}{f_{x,y}(\theta)} < \frac{1 - F_{x+1,y+1}(\theta)}{f_{x+1,y+1}(\theta)}$.

Proof *of* **Lemma 12**

(1) Generally the distribution function $F_{x,y}(\theta)$ and the probability density function $f_{x,y}(\theta)$ of the y th smallest order statistics from x i.i.d. random variables are given by:

$$1 - F_{x,y}(\theta) = \sum_{i=0}^{y-1} \binom{x}{i} F^i (1-F)^{x-i}, \tag{S1}$$

$$f_{x,y}(\theta) = \frac{x!}{(y-1)!(x-y)!} F^{y-1} (1-F)^{x-y} f. \tag{S2}$$

By calculation, we get

$$\frac{1 - F_{x,y}(\theta)}{f_{x,y}(\theta)} = \sum_{i=0}^{y-1} \frac{(y-1)!(x-y)!}{(x-i)!(i)!} (\frac{1-F(\theta)}{F(\theta)})^{y-i-1} \frac{1-F(\theta)}{f(\theta)}. \tag{S3}$$

(2) From the expression above we know that $\frac{1-F_{x,y}(\theta)}{f_{x,y}(\theta)}$ is strictly decreasing in θ provided $F(\cdot)$ satisfies the increasing-hazard-rate property and $y \geq 2$; When $y = 1$,

$$\frac{1 - F_{x,1}(\theta)}{f_{x,1}(\theta)} = \frac{1}{x} \frac{1-F(\theta)}{f(\theta)}, \tag{S4}$$

which is weakly decreasing in θ.

(3) To show $\frac{1-F_{x,y}(\theta)}{f_{x,y}(\theta)} > \frac{1-F_{x+1,y}(\theta)}{f_{x+1,y}(\theta)}$, we have

$\frac{1-F_{x,y}(\theta)}{f_{x,y}(\theta)} > \frac{1-F_{x+1,y}(\theta)}{f_{x+1,y}(\theta)}$

$\Leftrightarrow \sum_{i=0}^{y-1} \frac{(y-1)!(x+1-y)!}{(x+1-i)!(i)!}(\frac{1-F(\theta)}{F(\theta)})^{y-i-1}\frac{1-F(\theta)}{f(\theta)} < \sum_{i=0}^{y-1} \frac{(y-1)!(x-y)!}{(x-i)!(i)!}(\frac{1-F(\theta)}{F(\theta)})^{y-i-1}\frac{1-F(\theta)}{f(\theta)}$

$\Leftrightarrow \sum_{i=0}^{y-1} \frac{(x+1-y)}{(x+1-i)(x-i)!(i)!}(\frac{1-F(\theta)}{F(\theta)})^{y-i-1} < \sum_{i=0}^{y-1} \frac{1}{(x-i)!(i)!}(\frac{1-F(\theta)}{F(\theta)})^{y-i-1}$

$\Leftarrow \frac{(x+1-y)}{(x+1-i)(x-i)!(i)!} < \frac{1}{(x-i)!(i)!}, \forall 0 \leqslant i \leqslant y-1$

$\Leftrightarrow x+1-y < x+1-i, \forall 0 \leqslant i \leqslant y-1$

$\Leftrightarrow i < y, \forall 0 \leqslant i \leqslant y-1.$

(4) To show $\frac{1-F_{x,y}(\theta)}{f_{x,y}(\theta)}$ is strictly increasing in y, we only need to show $\frac{1-F_{x,y}(\theta)}{f_{x,y}(\theta)} < \frac{1-F_{x+1,y+1}(\theta)}{f_{x+1,y+1}(\theta)}$. Suppose it is true, we then have:

$\frac{1-F_{x,y}(\theta)}{f_{x,y}(\theta)} < \frac{1-F_{x+1,y+1}(\theta)}{f_{x+1,y+1}(\theta)} < \frac{1-F_{x,y+1}(\theta)}{f_{x,y+1}(\theta)}$.

To show $\frac{1-F_{x,y}(\theta)}{f_{x,y}(\theta)} < \frac{1-F_{x+1,y+1}(\theta)}{f_{x+1,y+1}(\theta)}$, we have

$$\begin{aligned}
\frac{1-F_{x+1,y+1}(\theta)}{f_{x+1,y+1}(\theta)} &= \sum_{i=0}^{y} \frac{y!(x-y)!}{(x+1-i)!(i)!}(\frac{1-F(\theta)}{F(\theta)})^{y-i}\frac{1-F(\theta)}{f(\theta)} \\
&= \sum_{i=1}^{y} \frac{y!(x-y)!}{(x+1-i)!(i)!}(\frac{1-F(\theta)}{F(\theta)})^{y-i}\frac{1-F(\theta)}{f(\theta)} + \frac{y!(x-y)!}{(x+1)!}(\frac{1-F(\theta)}{F(\theta)})^{y}\frac{1-F(\theta)}{f(\theta)} \\
&> \sum_{i=1}^{y} \frac{y!(x-y)!}{(x+1-i)!(i)!}(\frac{1-F(\theta)}{F(\theta)})^{y-i}\frac{1-F(\theta)}{f(\theta)} \\
&= \sum_{i=0}^{y-1} \frac{(y-1)!(x-y)!}{(x-i)!(i)!}(\frac{1-F(\theta)}{F(\theta)})^{y-i-1}\frac{1-F(\theta)}{f(\theta)}\frac{y}{(i+1)} \\
&\geq \frac{1-F_{x,y}(\theta)}{f_{x,y}(\theta)}. \text{ Q.E.D.}
\end{aligned}$$

We use Lemma 12 to show the following lemma.

Lemma 13. *Suppose* $\widetilde{F}(\theta) \triangleq \frac{F(\theta)}{F(\theta)}$ *when* $x \in [\underline{\theta}, k]$, *and* $\widetilde{F}_{N-y,N-q+1}$ *is the (N-q+1)th smallest order statistics among the* $(N-y)$*(with* $y \leq q-1$*) i.i.d. random variables with distribution* $\widetilde{F}(\theta)$*. We have:*

$\frac{1-\widetilde{F}_{N-y,N-q+1}(\theta;k)}{\widetilde{f}_{N-y,N-q+1}(\theta;k)}$ *is strictly increasing in* y*, decreasing in* θ*, strictly increasing in* k*, and continuously differentiable in* $(k,\theta) \in \{(k,\theta) : \underline{\theta} < \theta < k\}$*. If* $F(k) < 1$, $\frac{1-\widetilde{F}_{N-y,N-q+1}(\theta;k)}{\widetilde{f}_{N-y,N-q+1}(\theta;k)}$ *is strictly increasing in* θ*.*

Proof *of* **Lemma 13**

According to Lemma 12, $\frac{1-\widetilde{F}_{N-y,N-q+1}(\theta;k)}{\widetilde{f}_{N-y,N-q+1}(\theta;k)}$ is strictly strictly increasing in y. Given the expression of $\frac{1-\widetilde{F}_{N-y,N-q+1}(\theta;k)}{\widetilde{f}_{N-y,N-q+1}(\theta;k)}$,

$$\frac{1-\widetilde{F}_{N-y,N+1-q}(\theta;k)}{\widetilde{f}_{N-y,N+1-q}(\theta;k)} = \sum_{i=0}^{N-q} \frac{(N-q)![N-y-(N+1-q)]!}{(N-y-i)!(i)!} (\frac{F(k)-F(\theta)}{F(\theta)})^{N-q-i} \frac{F(k)-F(\theta)}{f(\theta)}, \tag{S5}$$

we can verify that it is strictly increasing and continuously differentiable in k. In the following we show that

$\frac{A-F(x)}{f(x)}$ is strictly decreasing for $F(x) < A, 0 < A < 1$, so that $\frac{1-\widetilde{F}_{N-y,N+1-q}(\theta;k)}{\widetilde{f}_{N-y,N+1-q}(\theta;k)}$is (strictly) decreasing in θ.

$(\frac{1-F}{f})' = \frac{-f^2-(1-F)f'}{f^2} \leq 0 \Rightarrow f' \geqslant -\frac{f^2}{1-F}$

$0 < A < 1 \Rightarrow 0 < A - F < 1 - F \Rightarrow -\frac{1}{1-F} > -\frac{1}{A-F}$.

Thus $f' \geqslant -\frac{f^2}{1-F} > -\frac{f^2}{A-F}$. As a result $(\frac{A-F}{f})' = \frac{-f^2-(A-F)f'}{f^2} < 0$. Therefore $\frac{1-\widetilde{F}_{N-y,N+1-q}(\theta;k)}{\widetilde{f}_{N-y,N+1-q}(\theta;k)}$is (strictly) decreasing in θ.

The continuous differentiability with respect to (k, θ) is obvious. Q.E.D.

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