Communication in Collective Bargaining*

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Abstract

We analyze how institutions shape incentives of communication in a legislative bargaining game. Legislators are privately informed about their ideal points in one-dimensional policy space. In one setting, cheap-talk communication precedes a take-it-or-leave-it agenda-setting game. The second involves sequential agenda setting in which the setter can revise the proposal only when the first one fails to gain enough support. The latter institution requires the setter to commit to a policy as a screening technology. The commitment fosters information disclosure from strategic voters and thus results in efficiency gains over straw polls, where the setter is not constrained in how she reacts to revealed information. With a focus on monotone equilibrium, we also find that simple majority rule sometimes induces no information disclosure in the cheap-talk stage, while unanimity rule always induces information disclosure.

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1 Introduction

1.1 Overview

The underlying logic of pivotal models (Brady and Volden, 1998; Krehbiel, 1996, 1998; Romer and Rosenthal, 1978, 1979) helps us to understand the general political bargaining process, especially with regard to the internal structure of legislative politics. Under institutional constraints, the agenda setter makes policy proposals in pursuit of her own goal while taking into account how other players respond to the proposals. In many situations when the setter faces uncertainty about others' preferences, communication with the other legislators helps her to learn their preferences and enhance targeting precision. However, political actors sometime withhold substantial information, and their incentives in the communication are shaped by the institutional arrangement. How does the institutional constraint that the setter faces affect the extent to which she can elicit information and utilize this information in policy making? Seemingly a more tightened constraint offers less flexibility to utilize information. However, by comparing two substantive institutional arrangements, our model shows that, tighter institutional constraints on the setter in equilibrium improve her ability to learn precisely because they constrain the ways she can use what she learns. Interestingly, the advantages for learning can offset the constraints on proposing, and thus make the setter better off.

We analyze two deliberative mechanisms in an agenda-setting environment with two periods. A setter faces two voters.[1] Each voter has private information about his ideal point.[2] The voting rule is simple majority in the benchmark model. No

[1] In the context of legislation, both the setter and the two voters are deemed as "legislators".

[2] Different from papers that mainly focus on the interaction between the setter and the voters (Banks, 1990, 1993; Lupia, 1992; Dewatripont and Roland, 1992; Morton, 1988), our paper con-

abstention is allowed.

One institutional arrangement allows voters to send cheap-talk messages simultaneously before the setter makes a take-it-or-leave-it offer. The status quo remains unchanged if the proposal is rejected collectively. We refer to this as a *straw poll*, or a *non-binding institution*. In an alternative setting, the setter is allowed to make an experimental proposal and commits to it if it is accepted. Upon the failure of the initial proposal, she can alter it and call for a re-vote. We refer to such a context with sequential agenda setting as a *binding institution*.[3]

We compare straw poll and binding votes, with the latter structure constraining the setter more. The comparison is conducted in two scenarios. First in a normative benchmark, we assume that voters are naive in the sense that they vote sincerely in the binding institution and respond truthfully to the binary question in the straw poll. We find that commitment makes the setter worse off. In this case, the binding institution serves as a special straw poll, which gives the setter a lower payoff than she would get if she designs the poll optimally. However, when we focus on equilibrium analysis which incorporates voters' strategic incentives for communication, less flexibility in using information allows better learning about voters' preferences and induces higher welfare for the proposer.[4]

As in Matthews (1989), the cheap-talk game we consider differs from the classical cheap-talk games (Crawford and Sobel, 1982; Green and Stokey, 2007) in two aspects. First, in classical cheap-talk games, the sender only has an indirect au-

siders the interaction between voters.

[3]This bargaining structure is common in the literature (Baron and Ferejohn, 1989; Banks and Duggan, 2000; Battaglini and Coate, 2005). It was used to study the public school budgeting in some areas of the United States (Romer and Rosenthal, 1979).

[4]The argument that commitment may benefit the decision maker is not new in the theory literature (e.g. Kydland and Prescott 1977). The contribution of the institutional comparison in our paper is to specify a substantive form of commitment in terms of real institutional arrangements in a political environment.

thority of influencing the policy through communication, and the receiver has an authority of policy making; whereas in our agenda-setting environment, the voters can not only indirectly influence the policy making through communication, but can also directly affect the policy in the process of a collective veto. Thus, our model applies to richer environments with different forms of authorities. Second, the content of communication in our model is the voters' private preferences, rather than a common fundamental that directly affects the setter's welfare.

Similar to the cheap talk with a single veto player (Matthews, 1989), under the straw poll with more than one voter, the types preferring the status quo and the types slightly preferring the setter's ideal point pool together in the communication and tend to induce some moderate proposal by sending signals of threat. The other types who sufficiently prefer setter's ideal point send signals of endorsement. Therefore, the equilibrium proposal of the setter depends on the percentage of the threat signals and endorsement signals. Whenever the number of endorsements satisfies the requirement of approval, the setter proposes her ideal policy and gets approved. In the other cases, compromise proposals will be induced. The degree of compromise depends on the number of voters sending the threat signal. Thus, the equilibrium outcome is equivalent to that of the situation when the setter first proposes her ideal point as a committed experiment. With enough endorsements, her ideal policy will be approved. Otherwise she has to make a compromise proposal. Accordingly, if the setter, under binding vote, proposes her ideal point, she can induce the same equilibrium lottery over policies as in the straw poll. Recall that the binding institution enables her to design an optimal initial proposal which may not be her ideal point. As a result, she gets a weakly higher payoff under the binding vote than under the straw poll.

In an extension, we also characterize equilibria when the voting rule is unanimity. Consistent with the main implication, when comparing different voting rules in cheap-talk communication, we find that simple majority rule which constrains the setter less sometimes induces no information transmission (in cut-point equilibria or even symmetric monotone equilibria, in which the types who send the same message form an interval[5]) in the communication stage, while unanimity rule always induces information disclosure. The result hinges on a simple intuition: a voter preferring the status quo could have an incentive to pretend to agree with the setter in communication. Under majority rule, a voter faces a trade-off between influencing the policy which might pass, and acting as a saboteur to mislead the setter who wants to enact policies that he likes less than the status quo. Under unanimity rule the voter can always block his less preferred policies, and thus has no incentive to fool the setter.

The remainder of the paper is organized as follows. Section 1.2 overviews the related literature. We introduce the basic setup and provide some preliminary results in Section 2. In Section 3, we compare the setter's welfare under the two institutions in a normative benchmark when the voters do not exhibit certain strategic consideration. In Section 4, we mainly study the existence of equilibria as well as connections between the binding vote and the non-binding straw poll under simple majority rule. In Section 5, we investigate several extensions and offer some discussions. Section 6 concludes.

[5]Our requirement of a monotone equilibrium is the same as the "connected equilibrium" in Chen and Eraslan (2014).

1.2 Literature Review

First, our paper contributes to the literature of collective bargaining with incomplete information.[6] Tsai and Yang (2010) study a majoritarian bargaining model where players have private information about their discount factors. Meirowitz (2007) studies how information can be fully revealed in collective bargaining with a common value setup. Agranov and Tergiman (2013a,b) and Baranski and Kagel (2013) use laboratory experiments to address the effect of communication in the Baron-Ferejohn environment. Our finding that more binding constraint on the setter improves information transmission, complements the result of Chen and Eraslan (2014). They show that two voters under simple majority may make the setter worse off than in the case with just a single voter with an absolute veto power. Specifically, Chen and Eraslan (2014) study three-player legislative bargaining with two dimensional policy (i.e., public policy and redistribution transfer) under simple majority rule before which simultaneous cheap-talk communication takes place. Voters have private information about the relative weights they put on ideological public policy content. Unlike their paper, we assume that the private information is about ideal points, and focus on public policy making without transfers. Chen and Eraslan (2013) consider a bargaining game similar to ours with straw poll. One important difference in the structure of the game is that they allow the setter to make redistributive transfers in addition to the ideological public policy.[7] They characterize the necessary conditions of the equilibrium, and compare the outcome of the bundled bargaining game in which the legislators negotiate over both issues

[6]For two-player bargaining with incomplete information, see Fudenberg et al. (1985) or Ausubel et al. (2002) for a survey. Chatterjee (2010) also provides a good survey and mentions the lack of enough studies on multilateral bargaining with incomplete information.

[7]Even when the budget of the whole "cake" is 0, because the setter's transfer to herself can be negative, by definition, the game in Chen and Eraslan (2013) is different from the game without the possibility of using private transfer.

together to that of the separate bargaining game in which the legislators negotiate over the issues one at a time. Alternatively, our primary goal is to compare a non-binding straw poll and a binding institution. In addition, we discuss the existence/non-existence of informative equilibrium with two voters/senders.

Our model is also related to recent work on how elections or committee decision are affected by communication or external influence (e.g. Felgenhauer and Peter Grüner 2008; Seidmann 2011). The comparison of different voting rules in the straw poll relates to the literature of strategic voting with communication (Austen-Smith and Feddersen, 2005, 2006; Gerardi and Yariv, 2007). For example, Austen-Smith and Feddersen (2005) show that in a partially common-value environment where deliberations precede the voting stage, majority rule induces more information sharing than unanimity in the deliberation stage. Similar to the sequential agenda-setting environment, in dynamic elections, the first-period communication in terms of cheap talk or vote totals aggregates relevant information and affects future collective decisions. For example, Piketty (2000), Razin (2003), Iaryczower (2008), Bond and Eraslan (2010) study the information transmission in a partially common-value voting setup. Messner and Polborn (2012) study voters' learning about their own preferences in an environment with sequentially collective decision making. In private-value environments, Shotts (2006) and Meirowitz and Shotts (2009) study how voters use the upstream election to signal their preferences and to indirectly affect future policy.

2 Model Setup

2.1 Institutional Arrangement

We mainly focus on a committee with $n = 2$ voters ($i = 1, 2$) and one setter. For simplicity, we assume that only the two voters can vote and the setter is not allowed to vote.[8] We first consider simple majority rule to illustrate the main insight. Proposal b will be collectively approved if and only if at least $q = 1$ committee member(s) vote(s) yes. Otherwise b will be rejected. In extensions, we also consider unanimity rule and other voting rules with an arbitrary number of voters.

We refer to b as the "budget", but b can be any public policy in general. We normalize the *status quo* s to be 0. The policy space is $[0, +\infty)$.[9]

In the *straw-poll* game, the **timing** is as follows:

Period (1): Each voter sends a binary[10] message to the setter;[11]

Period (2): The setter makes a proposal b_2. If the proposal is rejected, the status quo s will be implemented. Otherwise b_2 is implemented.

In the *binding vote* game, the **timing** is as follows:

Period (0): The setter makes an experimental proposal b_1;

Period (1): Each voter casts a vote on b_1. *If* b_1 *is* accepted, it is implemented, the game ends; and only if b_1 is rejected

Period (2): The setter makes a proposal b_2. If the proposal is rejected, the

[8]Under the straw poll, this assumption is equivalent to the assumption that all three players have voting rights. In the binding institution, the assumption rules out a possibility that the setter casts "nay" on her experimental proposal as a strategic choice.

[9]This assumption is made only for simplicity. The results will not change if we allow the policy to be "negative".

[10]In Section 5, we will relax the assumption of the binary message space.

[11]The message can be either public to voters or only privately observed by the setter. Both assumptions produce the same results.

status quo s will be implemented. Otherwise b_2 is implemented.

2.2 Preferences

Voter i's utility over policy x is

$$u_i(x;\theta_i) = 2\theta_i x - x^2, \tag{1}$$

with ideal point θ_i.[12]

Similarly, the setter with ideal point $\theta_A > 0$ has a utility over policy x.

$$u_A(x;\theta_A) = 2\theta_A x - x^2. \tag{2}$$

When θ_A becomes sufficiently large, the behavior is qualitatively similar to the classical Romer-Rosenthal model (Romer and Rosenthal, 1978, 1979), where the setter has a utility $u_A = x$. We assume players put equal weights on the two periods.

2.3 Information Structure

The private ideal point θ_i of each voter is i.i.d. drawn from an absolutely continuous distribution $F(\theta)$ with support $[0, \overline{\theta}]$. $\overline{\theta}$ is finite.[13]

The ideal point θ_i is voter i's private information. The distribution $F(\theta)$ and the setter's preference are common knowledge. We make the following assumption

[12] We make this assumption for simplicity. It is equivalent to assume $u_i(x;\theta_i) = -(\theta_i - x)^2$.

[13] We could allow the lower bound of the support $\underline{\theta}$ to be positive or negative. It will not change the main results. When $\underline{\theta} \geq 0$, we require that $2\underline{\theta} < \theta_A$, which is the condition that makes the setter want to learn voters' preferences. Otherwise, when $2\underline{\theta} \geq \theta_A$, she can just propose her ideal with unanimous approval. When $\underline{\theta} < 0$, for equilibrium analysis in the rest of the paper to be unaffected, we need to exclude the possible cut-point equilibrium $k \leq 0$. We can formally show that any cut-point $k \in (\underline{\theta}, 0]$ under a straw poll does not provide the setter "useful" information and gives her the same expected payoff as the pooling equilibrium.

about the composition of the committee, which is captured by the distribution function $F(\theta)$ and the setter's ideal point θ_A.

Assumption 1 *Committee Composition*

(1.1) $F(\cdot)$ *is twice continuously differentiable on* $[0, \overline{\theta})$; *(1.2) the probability density function* $f(\theta) > 0, \forall \theta \in [0, \overline{\theta})$; *(1.3) the hazard rate* $\frac{f(\theta)}{1-F(\theta)}$ *is weakly increasing for* $\theta \in [0, \overline{\theta})$; *and (1.4)* $0 < \theta_A \leq \overline{\theta}$.

The increasing hazard-rate condition in Assumption *1* is a standard condition in the Bayesian Game literature (Banks, 1993). Many distributions satisfy this assumption, such as the uniform distribution, (truncated) exponential distribution and normal distribution (Bergstrom and Bagnoli, 2005). We say that the setter is *moderate* if her ideal point θ_A is sufficiently close to the status quo 0.

2.4 Strategies and Equilibrium

Under the straw poll, there is no *Period (0)*. In *Period (0)* of the binding-vote game, the setter chooses an initial proposal $b_1 \in [0, +\infty)$.

In the communication stage of the straw poll, we assume that each voter's message space is binary, i.e., $\{1, 0\}$, so that each voter i's talking strategy is $T_i : [0, \overline{\theta}] \rightarrow \{1, 0\}$. In *Period (1)* of the binding-vote game, each voter i's voting strategy is $V_i^1 : [0, \overline{\theta}] \times [0, +\infty) \rightarrow \{1, 0\}$, where "1" represents a positive vote and "0" represents a negative vote. In other words, each voter's vote $v_i = V_i^1(\theta_i, b_1)$ depends both on his type and the initial proposal.

After observing all voters' messages (m_1, m_2) in the straw-poll game, the setter chooses a revised proposal $b_2 \in [0, +\infty)$. In the binding-vote game, only when the vote total $(v_1 + v_2)$ is less than $q = 1$, the setter chooses a revised proposal $b_2 \in [0, +\infty)$.

In the voting stage of the second period, we assume that voters vote *sincerely*. This assumption excludes the weakly dominated strategies in the final stage. Thus each voter votes for the revised proposal against the status quo if and only if $\theta_i \geq \frac{1}{2}b_2$. Formally in both games, the last stage voting strategy is

$$V^2(\theta_i, b_2) = \begin{cases} 1 & \text{if} \quad \theta_i \geq \frac{1}{2}b_2 \\ 0 & \text{if} \quad \theta_i < \frac{1}{2}b_2 \end{cases}. \tag{3}$$

The equilibrium notion is a (pure strategy) symmetric Perfect Bayesian *Nash Equilibrium.* Furthermore, we also focus on cut-point equilibria, i.e., equilibria in which each voter's strategy in the first period follows a cut-off rule with the cut-point k. Specifically, $T_i^*(\theta_i) = 1$ if an only if $\theta_i \geq k_S^*$; and $V_i^1(\theta_i, b_1) = 1$ if an only if $\theta_i \geq k_B^*(b_1)$, where $k_B^*(\cdot)$ is the equilibrium cut-point as a function the setter's experimental proposal b_1. Because we focus on symmetric cut-point equilibrium, the revised proposal b_2^* only depends on the number of endorsements y (i.e., claims or votes indicating that his ideal point θ_i is above the cut-point) instead of the whole profiles of messages (m_1, m_2) or votes (v_1, v_2).

The **equilibrium** in the straw poll $\{k_S^*,\ P_S^*(y),\ b_S^*(y),\ V^2(\theta_i, b_2)\}$ involves the following requirements:

(S-1) given the other voter's equilibrium cut-point k_S^*, the second-period equilibrium proposing strategy $b_S^*(y)$ as a function of the number of endorsements y, and the last stage sincere voting strategy $V^2(\theta_i, b_2)$, which is determined by equation (3), voter i with any type θ_i does not have an incentive to deviate from the cut-point rule k_S^* in the communication;

(S-2) $P_S^*(y)$ represents the setter's belief about voters' ideal points given k_S^* and y endorsements, and is derived by Bayes' rule whenever possible;

(S-3) the equilibrium proposal $b_S^*(y)$ needs to maximize the setter's welfare given her equilibrium belief about voters' ideal points $P_S^*(y)$; and

(S-4) $V^2(\theta_i, b_2)$ is the last stage sincere voting strategy, and is determined by equation (3).

Similarly, the **equilibrium** in the binding institution $\{b_1^*,\ k_B^*(b_1),\ P_B^*(y, b_1),\ b_B^*(y, b_1),\ V^2(\theta_i, b_2)\}$ involves the following requirements:

(B-0) given voters' equilibrium voting strategies $k_B^*(\cdot)$ and $V^2(\theta_i, b_2)$, as well as the revising-proposal strategy $b_B^*(y, b_1)$, b_1^* should maximize the setter's expected payoff;

(B-1) for any $b_1 \in [0, +\infty)$, given the other voter's equilibrium cut-point strategy $k^*(b_1)$, the second-period equilibrium proposing strategy $b_B^*(y, b_1)$, and the last stage sincere voting strategy $V^2(\theta_i, b_2)$, voter i with any type θ_i does not have an incentive to deviate from the cut-point strategy $k_B^*(b_1)$ in the first period voting;

(B-2) for any $b_1 \in [0, +\infty)$, $P_B^*(y, b_1)$ represents the setter's belief about voters' ideal points given $k_B^*(b_1)$, y endorsements and the initial proposal b_1, and is derived by Bayes' rule whenever possible;

(B-3) for any $b_1 \in [0, +\infty)$, the equilibrium proposal $b_B^*(y, b_1)$ needs to maximize the setter's welfare given her equilibrium belief about voters' ideal points $P_B^*(y, b_1)$; and

(B-4) $V^2(\theta_i, b_2)$ is the last stage sincere voting strategy, and is determined by equation (3).

For simplicity, we focus on the equilibrium with $k_B^*(b_1) > 0$ whenever $b_1 > 0$ in the binding-vote game.[14] If $k_B^*(b_1^*) \in (0, \overline{\theta})$ (or $k_S^* \in (0, \overline{\theta})$), we say that it is an informative equilibrium in the binding institution (or in the straw poll).

[14]This restriction excludes the possibility that every voter votes to accept the first proposal with probability 1 under the binding referendum.

2.5 Preliminary Results: Setter's Belief and the Second Period Proposal

In the second period, when the agenda setter makes her best proposal, she only needs to target the "pivotal" ideal point. Without loss of generality, we assume that $P_B^*(y, b_1)$ and $P_S^*(y)$ are only about the distribution of the "pivotal" ideal point (given voters' equilibrium cut-points), which we will specify as following.

Suppose that the setter belives that the voters use an interior cut-point k in the first period. Under the binding vote with $y = 0$, the setter targets the largest ideal point of the two i.i.d. draws from $\widetilde{F}(t_i; k) \triangleq \Pr(\theta_i \leqslant t_i \mid \theta_i \leqslant k)$, so that the targeted distribution is $\Omega(t_i|0; k) = \widetilde{F}(t_i; k)^2$. Under the straw poll with no endorsements, i.e., $y = 0$, the setter targets the same distribution as under the binding vote (providing that she believes that voters use the cut-point k), $\Omega(t_i|0; k) = \widetilde{F}(t_i; k)^2$. Upon only one endorsement, i.e., $y = 1$, the setter targets the ideal point that is larger than k, which has a distribution $\Omega(t_i|1; k) = \widehat{F}(t_i; k) \triangleq \Pr(\theta_i \leqslant t_i \mid k \leq \theta_i)$. With unanimous endorsements, i.e., $y = 2$, the setter targets the largest among two i.i.d. draws from $\widehat{F}(t_i)$, hence the targeted distribution is $\Omega(t_i|2; k) = \widehat{F}(t_i)^2$. By the construction of the function $\Omega(\cdot|y; k)$,[15] the equilibrium beliefs in informative equilibria can be directly recovered if we know the equilibrium cut-points k_S^* (or $k_B^*(b_1)$). Specifically,

$$P_S^*(y) = \{\Omega(\cdot|y; k_S^*)\}, \text{for } y = 0, 1, 2.^{16} \tag{4}$$

$$P_B^*(y, b_1) = \{\Omega(\cdot|y; k_B^*(b_1))\}, \text{ for } y = 0 \text{ and } \forall b_1 \in [0, +\infty).^{17} \tag{5}$$

[15]We define $\Omega(t_i|0; k = \overline{\theta}) = \widetilde{F}(t_i; k = \overline{\theta})^2$.

[16]If $k_S^* = \overline{\theta}$, we have $P_S^*(0) = \{\Omega(\cdot|0; k_S^* = \overline{\theta})\}$.

[17]If $k_B^*(b_1) = \overline{\theta}$, we have $P_B^*(0, b_1) = \{\Omega(\cdot|0; k_B^*(b_1) = \overline{\theta})\}$.

Based on the belief, we can derive the agenda setter's best response to the total number of endorsements y and her belief about voters' cut-point k. Because $[1 - \Omega(\frac{1}{2}b|y;k)]u_A(b)$ is the setter's expected payoff, her optimal proposal is then

$$\beta(y;k) \triangleq \underset{b\in[0,+\infty)}{\arg\max}[1 - \Omega(\frac{1}{2}b|y;k)]u_A(b), \tag{6}$$

By its definition, we can recover the revised-proposal strategy in informative equilibria, if we get the equilibrium cut-points, i.e.,

$$b_S^*(y) \in \beta(y;k_S^*), \text{ for } y = 0,1,2. \tag{7}$$

$$b_B^*(y,b_1) \in \beta(y;k_B^*(b_1)), \text{for } y = 0 \text{ and } \forall b_1 \in [0,+\infty). \tag{8}$$

Lemma 1 characterizes the properties of $\beta(y;k)$.

Lemma 1 *$\beta(y;k)$ defined in equation (6) represents the setter's optimal revised proposal(s) when she faces y endorsements or positive votes, and believes that voters use a symmetric cut-point strategy with the cut-point $k \in (0,\overline{\theta})$. Then,*

(1) $\beta(y;k)$ is single-valued, so that $b_S^*(y) = \beta(y;k^*)$, for $y = 0,1,2$, and $b_B^*(y,b_1) = \beta(y;k^*(b_1))$, for $y = 0$ and $\forall b_1 \in [0,+\infty)$;

(2) $\beta(y;k)$ is increasing in the total positive votes (or endorsements) of the communication stage, y;

(3) $\beta(y;k)$ is continuously differentiable (except for at most two points), continuous and increasing in k; and

(4) when $k \in (0,\frac{\theta_A}{2}), \theta_A > \beta(2;k) > \beta(1;k) \geq 2k > \beta(0;k)$; when $k \in [\frac{\theta_A}{2},\overline{\theta}), \theta_A = \beta(2;k) = \beta(1;k) > \beta(0;k)$.

The first part of the lemma suggests that the setter's best response to y and k is unique so that $\beta(y;k)$ is a well-defined function. Part (2) shows that the revised proposal must be increasing in the total endorsements, namely $\beta(2;k) \geq \beta(1;k) > \beta(0;k)$. This result comes from the assumption of symmetric cut-point in equilibrium and the fact that the setter prefers a higher budget than the status quo. Part (3) shows how the proposal $\beta(y;k)$ responds to the perceived cut-point k. Suppose that k increases. Given the same number of endorsements or positive votes, the setter tends to believe that the voters' ideal points will be higher in the probability sense. Thus, a slight increase in the budget proposal does not increase the failing rate, while it makes the setter enjoy a higher benefit from the policy when it is accepted. As a result, the setter's best response $\beta(y;k)$ is increasing in k. Part (3) implies that, when $k < \frac{\theta_A}{2}$, the setter always proposes compromise proposals, i.e., $\beta(y;k) < \theta_A$, and when $k \geq \frac{\theta_A}{2}$, the setter proposes her ideal point upon at least one endorsement, because in this case she knows for sure that the pivotal voter prefers her ideal point to the status quo.

2.6 Characterizing the Equilibrium Cut-Points by Payoff Gain Functions

Given the other voter's cut-point k, the utility difference between saying "yes" and "no" as a function of voter's ideal point θ_i is[18]

$$V_{diff}(\theta_i;k_S^*) = \begin{array}{c} (1-F(k_S^*))[V(\theta_i,\beta(2;k_S^*),1) - V(\theta_i,\beta(1;k_S^*),1)] \\ +F(k_S^*)[V(\theta_i,\beta(1;k_S^*),0) - V(\theta_i,\beta(0;k_S^*),0)]. \end{array} \tag{9}$$

Given the proposal in the second period b_2, $V(\theta_i,b_2,x)$ is the second-period continuation payoff of the voter with ideal point θ_i when the other voter says "yes" ($x=1$) or "no" ($x=0$) in communication. y, the total number of endorsements is

[18] Recall that $V_{diff}(\theta_i;k)$ is also a functional of functions $\beta(y;k)$. Since each $\beta(y;k)$ is a function of k, we can always write the payoff gain as a function of k and θ_i.

a piece of public information at the beginning of the second period. In addition to that, each voter also knows his private ideal point and the action he has taken in the first period. The private information creates heterogenous beliefs among voters about the pivotal ideal point and about which policy will be finally implemented.

Similarly, a voter's payoff gain function under the binding vote with simple majority rule is

$$V_{diff}(\theta_i; k_B^*(b_1), b_1) = F(k_B^*(b_1))[u(b_1; \theta_i) - V(\theta_i, \beta(0; k_B^*(b_1)), 0)]. \qquad (10)$$

Therefore, we have the following two observations.

Remark 1 *The equilibrium cut-point* $k_S^* \in (0, \overline{\theta}]$ *under the straw poll must satisfy the following conditions:*

(i) $V_{diff}(k_S^*; k_S^*) = 0$; *(ii)* $V_{diff}(\theta; k_S^*) \geq 0$, *whenever* $\theta \in (k_S^*, \overline{\theta}]$; *and (iii)* $V_{diff}(\theta; k_S^*) \leq 0$, *whenever* $\theta \in [0, k_S^*)$.

Remark 2 *The equilibrium cut-point function* $k^*(b_1) \in (0, \overline{\theta}]$ *under the binding vote must satisfy the following conditions:*

given any experimental proposal $b_1 \in [0, +\infty)$,*(i)* $V_{diff}(k_B^*(b_1); k_B^*(b_1), b_1) = 0$; *(ii)* $V_{diff}(\theta; k_B^*(b_1), b_1) \geq 0$, *whenever* $\theta \in (k_B^*(b_1), \overline{\theta}]$; *(iii)* $V_{diff}(\theta; k_B^*(b_1), b_1) \leq 0$, *whenever* $\theta \in [0, k_B^*(b_1))$.

3 Normative Benchmark: Optimized Poll with Naive Voters

We first provide a normative benchmark, namely a comparison of the setter's welfare under a straw poll and a binding vote when voters' behaviors are manipulatable in a certain way. Specifically, we assume that in a straw poll, the setter can choose a cut-point that maximizes her welfare, and voters "naively" use the cut-point designed by the setter to report individual preferences. In other words, we do not consider voters' incentives of communication as in Remark 1.

To make the comparison tractable, we also assume that they vote sincerely in the binding institution. That means they simply compare the payoffs under the experimental proposal and under the status quo. The cut-point is therefore $k = \frac{1}{2}b_1$.

Under the binding institution, the initial proposal is $b_1 = 2k$. If it gets passed, the pivotal ideal point must be higher than k. In this case, if the setter were allowed to revise the proposal, she can get better off, because she can always propose $b_1 = 2k$ again which will be passed with probability 1. If the initial proposal is rejected, she will propose the policy in the same way as in the straw poll given the same cut-point k. Therefore the setter can perform better by not committing to the initial proposal when it is accepted, providing cut-point k unchanged. Since she can flexibly design the binary poll (i.e., the cut-point) to her own favor, the optimized polling where she can impose the cut-point k for all voters, always gives her a (weakly) higher payoff than the sequential agenda setting.

We summarize the result in the following.

Proposition 1 *When voters are "naive" in the sense that they vote "sincerely" under the binding vote and respond truthfully to polls with a yes-no question designed by the setter, the optimized straw poll gives the setter a higher payoff than the binding referendum. (This result holds for any number of voters and any voting rule.)*

4 Main Results under Simple Majority Rule

4.1 Existence of Equilibrium under Straw Poll

We first characterize the necessary condition for an equilibrium under the straw poll. Suppose there exists a cut-point equilibrium such that $0 < k_S^* < \frac{1}{2}\theta_A$. By Lemma 1 we have $b_S^*(2) > b_S^*(1) \geq 2k_S^* > b_S^*(0)$. The equilibrium requires $V_{diff}(\theta_i = \frac{1}{2}b_S^*(1); k_S^*) \geq 0$. A voter with type $\frac{1}{2}b$ always gets 0 when the revised proposal is b. Hence $V(\theta_i = \frac{1}{2}b_S^*(1), b_S^*(1), 0) = V(\theta_i = \frac{1}{2}b_S^*(1), b_S^*(1), 1) = 0$, and

$$V_{diff}(\theta_i = \frac{1}{2}b_S^*(1); k_S^*) = (1-F(k_S^*))V(\theta_i = \frac{1}{2}b_S^*(1), b_S^*(2), 1) - F(k_S^*)V(\theta_i = \frac{1}{2}b_S^*(1), b_S^*(0), 0). \tag{11}$$

Since $b_S^*(2) > b_S^*(1) > b_S^*(0)$, we have $V(\theta_i = \frac{1}{2}b_S^*(1), b_S^*(2), 1) < 0$, $V(\theta_i = \frac{1}{2}b_S^*(1), b_S^*(0), 0) > 0$. Thus we get $V_{diff}(\theta_i = \frac{1}{2}b_S^*(1); k_S^*) < 0$, which contradicts our conjecture that $0 < k_S^* < \frac{1}{2}\theta_A$. As a result, any equilibrium cut-point $k_S^* \geq \frac{1}{2}\theta_A$. Hence, by Lemma 1, we get $b_S^*(2) = b_S^*(1) = \theta_A$. Accordingly, we simplify the payoff gain as

$$V_{diff}(\theta_i; k_S^*) = F(k_S^*)[V(\theta_i, \theta_A, 0) - V(\theta_i, b_S^*(0), 0)], \tag{12}$$

where the continuation payoffs $V(\theta_i, \theta_A, 0), V(\theta_i, b_S^*(0), 0)$ are characterized by equations (A8) and (A9) in the Appendix. We show the following result in the Appendix.

Proposition 2 *In the straw poll with simple majority rule,*

*(1) (**Necessary Conditions**) any cut-point equilibrium is greater than the sincere cut-point, i.e.,* $k_S^* \in (\frac{1}{2}\theta_A, \theta_A)$. k_S^* *is determined by* $k_S^* = \frac{1}{2}\theta_A + \frac{1}{2}\beta(0, k_S^*)$.

*(2) (**Sufficient Conditions**) a cut-point equilibrium exists, provided* $F(\cdot)$ *is convex on* $[0, \theta_A]$.

*(3) (**Non-Existence**) pooling is the unique cut-point equilibrium for sufficiently divergent committee. Specifically, for a sufficiently large $\overline{\theta}$, $\exists \delta > 0$ such that $\forall \theta_A \in (\overline{\theta} - \delta, \overline{\theta})$, pooling is the unique cut-point equilibrium,*[19] *provided $\lim_{\overline{\theta} \to +\infty} F(\cdot; \overline{\theta})$ has a finite variance.*[20]

The second part of Proposition 2 provides a sufficient condition for existence of the cut-point equilibrium. Any convex distribution function first-order stochastic dominates any concave distribution with the same support, provided they are both continuous. Hence, voters' ideal points are further away from the status quo 0 with a convex cumulative distribution function than with a concave cumulative distribution function.

The third part of Proposition 2 provides an analytical example for no information transmission with a divergent committee. If a cut-point equilibrium exists, a voter that prefers the status quo is supposed to say "no" according to the cut-point strategy. Nevertheless, he may pretend to induce a less compromising proposal in cheap talk. The proposal becomes more extreme than what the setter would propose if she had not received this additional "yea" message. This unreasonable proposal is more likely to be rejected. Once it is rejected, the voter will enjoy the benefit from the status quo. Yet by deviating, a voter also suffers a risk that the extreme proposal may be accepted by the other voter. The proposition implies that whether the "misleading" incentive kills the existence of the cut-point equilibrium, depends on the committee composition, i.e., the distribution of the voters' ideal points $F(\theta)$ and the setter's ideal point θ_A. When the committee is sufficiently divergent in a certain way, although we can find an indifferent type, as shown on the right-hand side of Figure 1, some low-demand types always want to

[19] In fact, we can show a stronger result that pooling is the unique cut-point equilibrium, even if we allow possibilities of asymmetric cut-points.

[20] For example, truncated exponential and normal distributions both satisfy the condition.

deviate. Technically, it means the single-crossing condition of continuation payoffs is violated.

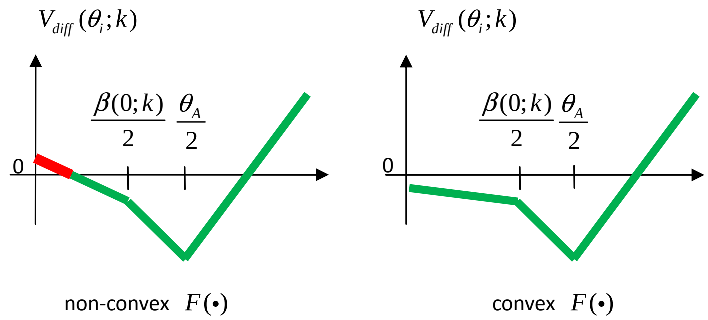

FIGURE 1. COMMUNICATION INCENTIVES WITH DIFFERENT COMMITTEE COMPOSITIONS

With a single veto player as in Matthews (1989), one can verify that a cut-point equilibrium always exists with the assumptions of our model. The proposition reflects a key difference between collective decision making and a single veto. When there is a single veto player (or collective decision making with unanimity rule as we show in the next section), each voter can directly veto a proposal he dislikes, so that it is not beneficial for him to mislead the setter. However, under simple majority, since each voter is uncertain about the other voter's preference and does not have a direct veto power, deviations may become profitable.

In general, it is difficult to prove the uniqueness of the cut-point equilibrium analytically. In the Supplementary Appendix we show that the cut-point equilibrium is unique with the uniform distribution.

4.2 Binding Vote and the Main Result about Replication

To compare the binding institution with the non-binding one, the following observation is useful.

Lemma 2 *If the straw-poll game has an equilibrium with the cut-point* $k_S^* \geq \frac{\theta_A}{2}$*, then there exists an informative equilibrium (i.e., an equilibrium with* $k_B^*(b_1^*) \in (0, \overline{\theta})$*) in the binding-vote game with the following properties:*

(1) with the equilibrium strategies (of the voters and the setter) other than the optimal initial proposal, if the setter chooses her ideal point as the initial proposal in the binding-vote game, she induces the same equilibrium distribution of outcome as in the straw-poll game, and $k_B^*(\theta_A) = k_S^*$*;*

(2) this equilibrium gives the setter a (weakly) higher payoff than that in the equilibrium k_S^* *of the straw poll; furthermore,*

(3) this equilibrium makes her strictly better off than in the case without any communication.[21]

(See the Appendix for the proof.)

The proof of this lemma is by construction. We first construct the equilibrium in the binding-vote game such that $k_B^*(\theta_A) = k_S^*$. The detailed construction of $k_B^*(b_1)$ when $b_1 \neq \theta_A$ is provided in the proof. This cut-point strategy requires that the voters use the cut-point k_S^* when the setter proposes her ideal point.

To verify that this strategy can be a part of an equilibrium, we need to show that no voter has an incentive to deviate from this voting strategy. In other words, we need to check the incentive compatibility constraints in Remark 2. Recall that voter's utility difference between saying "yes" and "no" in the communication stage of the straw poll, is $V_{diff}(\theta_i; k_S^*)$. The expression is determined by equation (12). A voter's utility difference between voting "yes" and "no" in the first period of the binding institution, is $V_{diff}(\theta_i; k_B^*(\theta_A) = k_S^*, b_1 = \theta_A)$, providing that the initial proposal is the setter's ideal point, and that the other voter's cut-point is $k_B^*(\theta_A)$. The expression is determined by equation (10). The two payoff gains capture a

[21]The result holds with any number of voters and any voting rule.

voter's communication incentives under the two institutions. We draw them in Figure 2.

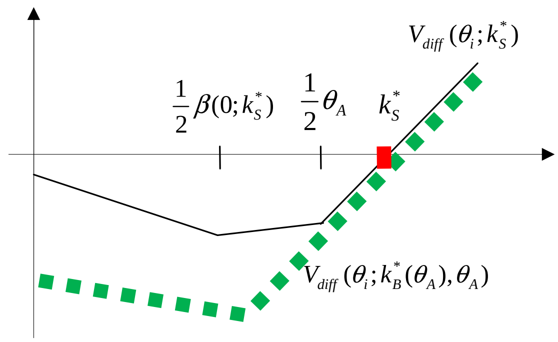

FIGURE 2. COMPARING INCENTIVES UNDER DIFFERENT INSTITUTIONS

As is shown in Figure 2, we can verify that the two payoff gains satisfy the following conditions (provided $k_S^* \geq \frac{1}{2}\theta_A$):

(a) $V_{diff}(\theta_i; k_S^*) = V_{diff}^{bind}(\theta_i; k_B^*(\theta_A) = k_S^*, b_1 = \theta_A)$ when $\theta_i \geq \frac{1}{2}\theta_A$; and

(b) $V_{diff}(\theta_i; k_S^*) \geq V_{diff}^{bind}(\theta_i; k_B^*(\theta_A) = k_S^*, b_1 = \theta_A)$ when $\theta_i < \frac{1}{2}\theta_A$.

Condition (a) suggests that the two functions have the same zero point, i.e., cut-point k_S^*, and a voter has the same incentive under the two institutions whenever his ideal point is above $\frac{1}{2}\theta_A$. Condition (b) suggests that for the voter with ideal point smaller than $\frac{1}{2}\theta_A$, if he casts a "negative" message in the straw poll, he also casts a "negative" vote in the first period of the binding institution. As a result, the type $\theta_i \leq k_B^*(\theta_A) = k_S^*$ does not have an incentive to deviate from the equilibrium voting strategy. Hence, no voter with any type has an incentive to deviate from the voting strategy $k_B^*(\theta_A) = k_S^*$.

In the straw poll, upon receiving at least one endorsement, the setter proposes her ideal point, and gets unanimous approval. In the other case, upon no endorsements, the setter makes some compromise proposal and bears the risk of being rejected. This outcome in the straw poll looks as if she sets the experimental proposal b_1 to be her ideal point in the binding institution, then all voters strategically vote between her ideal point and the status quo. Because $k_B^*(\theta_A) = k_S^* \geq \frac{1}{2}\theta_A$, the induced proposal $b_B^*(0, \theta_A)$ in the binding institution when the initial one gets rejected is the same as $b_S^*(0)$, which is the one in the straw poll when there is no endorsement. As a result, it induces an equilibrium stochastic outcome that is the same as in the straw poll. In general, the setter could possibly choose any policy other than her ideal point if that policy induces higher expected welfare. So the binding institution could always generate an equilibrium which gives the setter a weakly higher payoff than k_S^* equilibrium in the straw poll.

Based on Lemma 2, in order to explore the relationship between the two institutions, we only need to pin down the conditions under which the straw poll induces equilibria such that $k^* \geq \frac{1}{2}\theta_A$. An implication of Proposition *2* is that under simple majority in the straw poll, we always have $k_S^* > \frac{1}{2}\theta_A$. Thus we get the following proposition.

Proposition 3 *Under simple majority rule, the binding referendum has an informative equilibrium (i.e., an equilibrium with $k_B^*(b_1^*) \in (0, \overline{\theta})$) that (weakly) dominates any equilibrium in polling institution in terms of setter's welfare; in addition, this equilibrium also gives the setter a strictly higher payoff than in the static one-period model without learning.*[22]

(See the Appendix for the proof.)

The result contrasts with the normative benchmark in Section 3, that commitment hurts the setter's welfare when voters naively respond to the optimized poll.

[22]The result does not depend on if the straw-poll game has an informative equilibrium or not. Even when there is no cut-point equilibrium under a straw poll, the setter's ideal point, as one experimental proposal, always induces informative (sub-form) cut-point equilibrium under a binding vote.

5 Extensions

For the first two subsections, we will omit a full description of the extensive form of the game and the conditions for the equilibrium $\{k_S^*, P_S^*(y), b_S^*(y), V^2(\theta_i, b_2)\}$ in the straw-poll game and the equilibrium $\{b_1^*, k_B^*(b_1), P_B^*(y, b_1), b_B^*(y, b_1), V^2(\theta_i, b_2)\}$ in the binding-vote game, because they are analogous to those with two voters and simple majority rule.

5.1 Unanimity Rule

In this part, we extend the analysis under simple majority rule to unanimity rule. Different from the property in simple majority rule that a cut-point equilibrium may not exist, unanimity rule always results in equilibria with information transmission in cut-point strategies, because in unanimity rule each voter has a veto power. Whenever one dislikes the proposal, he can veto it without suffering from the cost of misleading the setter. Therefore, we have

Proposition 4 *In the straw poll with unanimity rule, a cut-point equilibrium* $k_S^* \in (0, \overline{\theta})$ *exists.*

(See the Appendix for the proof.)

Unanimity rule constrains the setter more than simple majority rule. This induces more information transmission in the deliberation. The comparison between the rules is consistent with our first main result, that more political constraints in terms of commitment elicit more information. In the following, we characterize a qualitative feature the equilibrium cut-point k_S^* under unanimity rule. Slightly different from majority rule, for sufficiently polarized committee, it is possible that $k_S^* < \frac{1}{2}\theta_A$ and the setter will never propose her ideal policy in this cheap-talk game.

Lemma 3 *In the straw poll with unanimity rule,*

(1) as long as one of the following conditions holds, any equilibrium cut-point $k_S^* > \frac{\theta_A}{2}$*:*

[1.1] $\theta_A \leq 2F^{-1}(\frac{1}{2})$ *or* $\overline{\theta} \leq 2F^{-1}(\frac{1}{2})$*; [1.2]* $F(\cdot)$ *is convex; and*

(2) any equilibrium cut-point is lower than the sincere cut-point, i.e., $k_S^* < \frac{1}{2}\theta_A$*, if the committee is sufficiently divergent in a certain way. Specifically, for sufficiently large* $\overline{\theta}$*,* $\exists M_2 < \overline{\theta}$*, s.t. whenever* $\theta_A \in (M_2, \overline{\theta})$*, any equilibrium cut-point* $k_S^* < \frac{1}{2}\theta_A$*, provided* $\lim_{\overline{\theta} \to +\infty} F(\theta; \overline{\theta})$ *exists; in this case, the setter always proposes compromising proposals, i.e.,* $b_2^*(\cdot) < \theta_A$*.*

In the straw poll with unanimity rule, when the setter is sufficiently moderate, or the committee is sufficiently homogeneous, the equilibrium cut-point is greater than the sincere cut-point, i.e., $k_S^* \geq \frac{1}{2}\theta_A$. However, for a sufficiently polarized committee and a polarized setter, $k_S^* < \frac{1}{2}\theta_A$. A direct implication of $k_S^* < \frac{1}{2}\theta_A$ is that the setter's proposals are always compromising, i.e., $b_2^*(\cdot) < \theta_A$. This contrasts with the equilibria in models with a single veto player (Matthews, 1989), where non-compromising proposal (θ_A) is induced with a positive probability in an informative equilibrium. In our model, with a single veto player, the equilibrium cut-point is always higher than the sincere cut-point $\frac{1}{2}\theta_A$, and whenever the voter indicates that his ideal point is above that cut-point, the setter proposes her ideal point θ_A.

Under the conditions that make $k_S^* \geq \frac{\theta_A}{2}$, we can directly apply Lemma 2. Similarly as with majority rule, voters behave in the same way in the binding institution with $b_1 = \theta_A$, as in the straw poll. So does the setter when she receives less than 2 endorsements, therefore we have the following claim.

Proposition 5 *Under unanimity rule, suppose at least one of the three conditions is satisfied:* $\theta_A \leq 2F^{-1}(\frac{1}{2})$ *or* $\overline{\theta} \leq 2F^{-1}(\frac{1}{2})$*, or* $F(\cdot)$ *is convex. Then, the binding ref-*

erendum has an informative equilibrium (i.e., an equilibrium with $k_B^*(b_1^*) \in (0, \overline{\theta})$) that (weakly) dominates any equilibrium in polling institution in terms of setter's welfare; in addition, this equilibrium also gives the setter a strictly higher payoff than in the static one-period model without learning.

5.2 A Generalized Result

In many situations, some supermajority rules are used instead of the unanimity rule. In the following, we generalize our result to an arbitrary number of voters and an arbitrary voting rule.

Proposition 6 *With n voters and q rule ($n \geq q \geq 1$), suppose an informative equilibrium exists in the straw poll. If the setter is sufficiently moderate (i.e., θ_A is sufficiently close to the status quo,0), then*

(1) in any equilibrium of the straw-poll game, $k_S^* \geq \frac{1}{2}\theta_A$;

(2) hence, the binding referendum has an informative equilibrium (i.e., an equilibrium with $k_B^*(b_1^*) \in (0, \overline{\theta})$) that (weakly) dominates any equilibrium in polling institution in terms of setter's welfare; in addition, this equilibrium also gives the setter a strictly higher payoff than in the static one-period model without learning.

(See the Supplementary Appendix for the proof.)

Notice that the proposition does not say anything about the existence of the cut-point equilibrium in the straw poll. Instead, it says, if a cut-point equilibrium exists under the straw poll, we can construct an equilibrium in the binding-vote game that (weakly) dominates the cut-point equilibrium in polling. The key of the proof is to show that the straw poll only induces information structures with $k_S^* \geq \frac{1}{2}\theta_A$, so that we can directly apply Lemma 2.

5.3 Richer Message Space in Straw-Poll Game

In the straw poll we focused on the equilibrium with binary message space. Now we consider an extended straw-poll model, where we do not impose any restriction on the message space and the voters can send as many messages as they want. The equilibrium notion is still a (pure strategy) symmetric Perfect Bayesian *Nash Equilibrium.* In addition, we focus on *monotone equilibria*, in which the sets of the separated types can be well ordered, and specifically the types that report the same message form an interval.

In the following proposition we show that the only possible equilibrium with monotone strategies in the straw poll is the cut-point equilibrium. In other words, each voter sends, at most, two different informative messages, although the message space is large.

Proposition 7 *In the straw-poll game with 2 voters and q rule ($q = 1, 2$), cut-point equilibrium is the only possible symmetric monotone equilibrium if one of following conditions holds:*

(1) q=1, (i.e., simple majority rule); (2) q=2, (i.e., unanimity rule) and $F(\cdot)$ is convex; and (3) q=2, (i.e., unanimity rule) and $\theta_A \leq 2F^{-1}(\frac{1}{2})$.

We prove the proposition in the Supplementary Appendix. The basic idea of the proof is to use the incentive compatibility condition of the lowest indifferent type. The result of the cut-point equilibrium comes from the specific distribution of authorities in the bargaining structure.

6 Conclusion

In this paper, we take a first step to develop a committee communication model within the context of Romer-Rosenthal agenda setting with private values and private information. The comparison between the two mechanisms illustrates the interaction between commitment and communication incentives. We address a fundamental question about how the political constraint the setter faces affects information transmission from strategic voters. The model shows that more commitment could improve the setter's ability to learn, precisely because it constrains the ways she can use what she learns. The advantages to learning can offset the constraints on proposing and make the setter better off. In the Supplementary Appendix, we also investigate how the voters' behaviors ("naive" v.s. "sophisticated") in the straw poll determine different levels of information disclosure and their welfare implications for the setter and voters. Consistent with the main finding, we also find that voting rules (as different political constraints) play different roles in eliciting information. In contrast with the traditional view, we find that unanimity rule may transmit more information in deliberation than simple majority rule if we focus on symmetric monotone equilibrium.

The model not only provides new insights into the classical cheap-talk model with a single veto player, but also contributes to the literature of political communication with private values. It is worthwhile to compare our model with Meirowitz and Shotts (2009), since the comparison reflects the difference between a "monopoly" model and a "competitive" model. In Meirowitz and Shotts (2009), the second-period policy converges to the median voter's ideal point because there is competition between the two candidates. As a result, the single-crossing condition is automatically satisfied. In our model, the setter has a monopoly proposing power because of no competition. Every voter uses private information to infer the likelihood that the revised proposal will be implemented. The uncertainty about whether the other voter will accept the proposal or not creates an incentive for a voter preferring the status quo to fool the policy maker in the "wrong" direction. In a separate but related paper, Chen (2013) characterizes the existence of an informative equilibrium under sequential agenda setting with an arbitrary number of voters and arbitrary voting rule, although the single-crossing condition is violated. However, whether an informative equilibrium exists and what properties the equilibria have (if any) in the generalized model with cheap-talk remains unknown.

Appendix

With n voters and q-rule, we omit a full description of the extensive form of the game and the conditions for the equilibrium $\{k_S^*,\ P_S^*(y),\ b_S^*(y),\ V^2(\theta_i, b_2)\}$ in the straw-poll game and the equilibrium $\{b_1^*,\ k_B^*(b_1),\ P_B^*(y, b_1),\ b_B^*(y, b_1),\ V^2(\theta_i, b_2)\}$ in the binding-vote game, because they are analogous to those with two voters and simple majority rule.

Defining and Characterizing the Setter's Belief and the Best Response

With n voters and q-rule, suppose that the setter believes that voters use the cut-point $k \in (0, \overline{\theta}]$, and observes y, the number of endorsements or positive votes (i.e., the signals indicating their ideal points are above the cut-point). We first define the conditional distribution of the pivotal ideal point $\Omega(t_i|y;k)$.[23]

$\widetilde{F}(t_i;k) \triangleq \Pr(\theta_i \leqslant t_i \mid \theta_i \leqslant k)$ (which is $\frac{F(t_i)}{F(k)}$ when $t_i \in [\underline{\theta}, k]$) is the setter's updated belief about voter i's ideal point if i casts a negative vote/message in the first period. $\widetilde{F}_{r,t}(t_i;k)$ denotes the distribution of the t th smallest order statistics from the r i.i.d. random variables $\theta_i \mid_{\theta_i \leqslant k}$.

For less than q supports (i.e. $y < q$), the setter targets the $(n-q+1)$ th smallest ideal point among the $(n-y)$ ideal points as the pivotal one. Thus

$$\Omega(t_i|y;k) \triangleq \widetilde{F}_{n-y,n-q+1}(t_i;k), \text{ for } y < q, k \in (0, \overline{\theta}), \text{ or for } y = 0, k = \overline{\theta}. \qquad \text{(A1)}$$

If there are q or more than q endorsements, i.e. $y \geq q$, the setter needs to target the $(y - q + 1)$ th smallest ideal point among the y ideal points with i.i.d.

[23] $\Omega(t_i|y;k)$ should depend on n and q.

distribution $\widehat{F}(t_i;k) \triangleq \Pr(\theta_i \leqslant t_i \mid k \leq \theta_i)$ (which is $\frac{F(t_i)-F(k)}{1-F(k)}$when $t_i \in [k,\overline{\theta}]$). We denote this targeted pivotal distribution as $\widehat{F}_{y,y-q+1}(t_i;k)$. So we have

$$\Omega(t_i|y;k) \triangleq \widehat{F}_{y,y-q+1}(t_i;k), \text{ for } y \geq q, k \in (0,\overline{\theta}). \tag{A2}$$

Whenever $\Omega(t_i|y;k)$ is well defined, the setter's best response is therefore

$$\beta(y;k) \triangleq \underset{b\in[0,+\infty)}{\arg\max}[1-\Omega(\frac{1}{2}b|y;k)]u_A(b). \tag{A3}$$

We characterize $\beta(y;k)$ in the following lemma, leaving the proof in the Supplementary Appendix.

Lemma 4 *(Full Characterization of the Second-Period Proposal) Suppose k is the cut-point that the setter believes that voters use, and y is the number of endorsements or positive votes. The second-period proposal $\beta(y;k)$ is characterized as follows:*

(1) for $y \leq q-1$,

$\beta(y;k) \in (0,\min\{2k,\theta_A\})$ and $\beta(y;k)$ is uniquely determined by

$$\frac{1-\widetilde{F}_{y,n+1-q}(\frac{1}{2}\beta;k)}{\widetilde{f}_{y,n+1-q}(\frac{1}{2}\beta;k)} = \frac{1}{2}\frac{u_A(\beta)}{u'_A(\beta)}; \tag{A4}$$

(2) for $y \geq q$,

(2.1) when $0 < \frac{1}{2}\theta_A \leq k < \overline{\theta}$, we have $\beta(y;k) = \theta_A$; and

(2.2) when $0 < k < \frac{1}{2}\theta_A$, we have $\lim_{k\to(\frac{1}{2}\theta_A)^-}\beta(y;k) = \theta_A$; furthermore

if $y = q$ and $k \geq \widehat{k}$, where $\widehat{k}$ is uniquely determined by $\frac{1}{q}\frac{1-F(k)}{f(k)} = \frac{1}{2}\frac{u_A(2k)}{u'_A(2k)}$, we have $\beta(y;k) = 2k$;

otherwise, we have $\beta(y;k) \in (2k, \theta_A)$, and $\beta(y;k)$ is uniquely determined by

$$\frac{1-\widehat{F}_{y,y-q+1}(\frac{1}{2}\beta;k)}{\widehat{f}_{y,y-q+1}(\frac{1}{2}\beta;k)} = \frac{1}{2}\frac{u_A(\beta)}{u'_A(\beta)}.^{24} \tag{A5}$$

Lemma 5 *Suppose k is the cut-point that the setter believes that voters use, and y is the number of endorsements or positive votes. We have*

(1) the revised proposal $\beta(y;k)$ is unique,[25] *so that $b^*_S(y) = \beta(y;k^*_S)$, for $y = 0, 1, ..., n$, where k^*_S is the equilibrium cut-point in the straw-poll game and $b^*_B(y, b_1) = \beta(y;k^*_B(b_1))$, for $y < q$ and $b_1 \in [0, +\infty)$, where $k^*_B(b_1)$ is the equilibrium cut-point in the binding-vote game;*

(2) $\beta(y;k)$ is increasing in the total positive votes (or endorsements) of the communication stage y; when $y \leq q$ or $k < \frac{1}{2}\theta_A$, $\beta(y;k)$ is strictly increasing in y;

(3) $\beta(y;k)$ is continuously differentiable (except for, at most, two points), continuous and increasing in k.[26]

Proof *of* **Lemma 5**

(1) According to the full characterization in Lemma 4, we know that $\beta(y;k)$ is uniquely determined. Hence by definition, we have $b^*_S(y) = \beta(y;k^*_S)$, for $y = 0, 1, ..., n$, where k^*_S is the equilibrium cut-point in the straw-poll game and $b^*_B(y, b_1) = \beta(y;k^*_B(b_1))$, for $y < q$ and $b_1 \in [0, +\infty)$, where $k^*_B(b_1)$ is the equilibrium cut-point in the binding-vote game.

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[24] If $y = q$, we have $\frac{1-\widehat{F}_{y,y-q+1}(\frac{1}{2}\beta;k)}{\widehat{f}_{y,y-q+1}(\frac{1}{2}\beta;k)} = \frac{1-F_{y,y-q+1}(\frac{1}{2}\beta;k)}{f_{y,y-q+1}(\frac{1}{2}\beta;k)}$.

[25] $\beta(y;k)$ is well defined except for when $k = \overline{\theta}$ (every voter casts a negative claim or vote) and $y \geq 1$ (the setter sees some positive claims or votes). The proposals in these cases depend on the off-the-equilibrium-path beliefs.

[26] When $y \leq q-1$, $\beta(y;k)$ is strictly increasing and continuously differentiable in k. When $y > q-1$, $\beta(y;k)$ has, at most, two kinks. When k is between the two kinks, $\beta(y;k)$ is strictly increasing and continuously differentiable in k; when k is outside of the interval bounded by the two kinks, $\beta(y;k)$ is constant within each of the two intervals.

(2)

(2.1) When $y \le q-1$, we know that $\beta(y;k) \in (0, \min\{2k, \theta_A\})$ and is uniquely determined by

$$\frac{1-\widetilde{F}_{y,n+1-q}(\frac{1}{2}\beta;k)}{\widetilde{f}_{y,n+1-q}(\frac{1}{2}\beta;k)} = \frac{1}{2}\frac{u_A(\beta)}{u_A'(\beta)}. \tag{A6}$$

The left-hand side of the first-order condition is the inverse of the hazard rate function of the corresponding order statistics, which is a strictly decreasing function of β, as shown in Lemma 7 of the Supplementary Appendix. The right-hand side of the first-order condition $\frac{1}{2}\frac{u_A(\beta)}{u_A'(\beta)}$, is a strictly increasing function in β when $\beta \in [0, \theta_A]$. Thus $\beta(y;k)$ is determined by the interaction of a decreasing curve $\frac{1-\widetilde{F}_{y,n+1-q}(\frac{1}{2}\beta;k)}{\widetilde{f}_{y,n+1-q}(\frac{1}{2}\beta;k)}$ and an increasing curve $\frac{1}{2}\frac{u_A(\beta)}{u_A'(\beta)}$. Because $\frac{1-\widetilde{F}_{y,n+1-q}(\frac{1}{2}\beta;k)}{\widetilde{f}_{y,n+1-q}(\frac{1}{2}\beta;k)}$ is strictly increasing in y by Lemma 7 in the Supplementary Appendix, as the parameter y in this decreasing function increases, the curve of $\frac{1-\widetilde{F}_{y,n+1-q}(\frac{1}{2}\beta;k)}{\widetilde{f}_{y,n+1-q}(\frac{1}{2}\beta;k)}$ moves upward so that its intersection with $\frac{1}{2}\frac{u_A(\beta)}{u_A'(\beta)}$ becomes larger, therefore $\beta(y;k)$ is strictly increasing in y whenever $y \le q-1$. Because $\beta(q-1;k) < 2k \le \beta(q;k)$, $\beta(y;k)$ is strictly increasing in y whenever $y \le q$.

(2.2) When $y \ge q$ and $\frac{1}{2}\theta_A \le k \le \overline{\theta}$, we have $\beta(y;k) = \theta_A$.

(2.3) Now suppose, $y \ge q$, $k < \frac{1}{2}\theta_A$, and $\frac{1}{q}\frac{1-F(k)}{f(k)} > \frac{1}{2}\frac{u_A(2k)}{u_A'(2k)}$. In this case, $\beta(y;k)$ is determined by the first order condition. Because $\frac{1-\widehat{F}_{y,y-q+1}(\frac{1}{2}\beta;k)}{\widehat{f}_{y,y-q+1}(\frac{1}{2}\beta;k)}$ is strictly increasing in y by Lemma 7 in the Supplementary Appendix, using the similar logic in (2.1), we know that $\beta(y;k)$ is strictly increasing in y whenever $y \ge q$.

(2.4) Now suppose $y \ge q$, $k < \frac{1}{2}\theta_A$, and $\frac{1}{q}\frac{1-F(k)}{f(k)} \le \frac{1}{2}\frac{u_A(2k)}{u_A'(2k)}$ (i.e., $k \ge \widehat{k}$). In the same way of (2.1), we can show that $\beta(y;k)$ is strictly increasing in y whenever $y \ge q+1$. Furthermore we also know that $\beta(q+1;k) > 2k = \beta(q;k)$. Thus $\beta(y;k)$ is strictly increasing in y whenever $y \ge q$.

(2.5) Combining the results in (2.1)-(2.4), we know that $\beta(y;k)$ is increasing in

y. In addition, when $y \le q$ or $k < \frac{1}{2}\theta_A$, $\beta(y;k)$ is strictly increasing in y;

(3)

(3.1) When $y \le q-1$, we know that $\beta(y;k) \in (0, \min\{2k, \theta_A\})$ and is uniquely determined by

$$\frac{1-\widetilde{F}_{y,n+1-q}(\frac{1}{2}\beta;k)}{\widetilde{f}_{y,n+1-q}(\frac{1}{2}\beta;k)} = \frac{1}{2}\frac{u_A(\beta)}{u_A'(\beta)}. \tag{A7}$$

Because $\frac{1-\widetilde{F}_{y,n+1-q}(\frac{1}{2}\beta;k)}{\widetilde{f}_{y,n+1-q}(\frac{1}{2}\beta;k)}$ is strictly increasing in k by Lemma 7 in the Supplementary Appendix**,** as the parameter k in this decreasing curve increases, the curve of $\frac{1-\widetilde{F}_{y,n+1-q}(\frac{1}{2}\beta;k)}{\widetilde{f}_{y,n+1-q}(\frac{1}{2}\beta;k)}$ moves upward so that its intersection with $\frac{1}{2}\frac{u_A(\beta)}{u_A'(\beta)}$ becomes larger, therefore $\beta(y;k)$ is strictly increasing in k whenever $y \le q-1$. In addition, the continuous differentiability directly comes from the fact that $\frac{1-\widetilde{F}_{y,n+1-q}(\frac{1}{2}\beta;k)}{\widetilde{f}_{y,n+1-q}(\frac{1}{2}\beta;k)}$ and $\frac{1}{2}\frac{u_A(\beta)}{u_A'(\beta)}$ are twice continuously differentiable in β and k.

(3.2) When $y > q$, and $0 < k < \frac{1}{2}\theta_A$, we can show the same results in the same way.

When $y > q$ and $k \ge \frac{1}{2}\theta_A$, we have $\beta(y;k) = \theta_A$.

Because $\lim_{k \to (\frac{1}{2}\theta_A)^-} \beta(y;k) = \theta_A$ by Lemma 4, we know that $\beta(y;k)$ is continuously differentiable (except for at $k = \frac{1}{2}\theta_A$), continuous and increasing in k. $\beta(y;k)$ is strictly increasing in k when $k < \frac{1}{2}\theta_A$.

(3.3) Now suppose $y = q$.

When $k \in [\frac{1}{2}\theta_A, \overline{\theta})$, we have $\beta(q;k) = \theta_A$.

When $k \in [\widehat{k}, \frac{1}{2}\theta_A)$,[27] we have $\beta(q;k) = 2k$.

When $k \in [0, \min\{\frac{1}{2}\theta_A, \widehat{k}\})$, $\frac{1-\widehat{F}_{y,y-q+1}(\frac{1}{2}\beta;k)}{\widehat{f}_{y,y-q+1}(\frac{1}{2}\beta;k)}$ is independent of k so that $\beta(q;k)$ does not depend on k.

Therefore $\beta(y;k)$ has, at most, two kinks. When k is between the two kinks, $\beta(y;k)$ is strictly increasing and continuously differentiable in k. When k is outside

[27] If $\widehat{k} \ge \frac{1}{2}\theta_A$, we define $[\widehat{k}, \frac{1}{2}\theta_A) = \emptyset$.

of the interval bounded by the two kinks, $\beta(y;k)$ is constant within each of the two intervals. Q.E.D.

$$V(\theta_i,\theta_A,0)=\begin{cases}2\theta_i\theta_A-\theta_A^2 & \text{if } \theta_i\geq\frac{1}{2}\theta_A\\ [1-\widetilde{F}(\frac{1}{2}\theta_A;k_S^*)][2\theta_i\theta_A-\theta_A^2] & \text{if } \theta_i<\frac{1}{2}\theta_A\end{cases}, \tag{A8}$$

$$V(\theta_i,b_S^*(0),0)=\begin{cases}2\theta_i b_S^*(0)-b_S^*(0)^2 & \text{if } \theta_i\geq\frac{1}{2}b_S^*(0)\\ [1-\widetilde{F}(\frac{1}{2}b_S^*(0);k_S^*)][2\theta_i b_S^*(0)-b_S^*(0)^2] & \text{if } \theta_i<\frac{1}{2}b_S^*(0)\end{cases}. \tag{A9}$$

Proof *of* **Proposition** 2

(1)From equations (A8) and (A9), we get

$$\frac{V_{diff}(\theta_i;k_S^*)}{F(k_S^*)}=$$

$$\begin{cases}[2\theta_i\theta_A-\theta_A^2]-[2\theta_i b_S^*(0)-b_S^*(0)^2] & \text{if } \theta_i\in[\frac{1}{2}\theta_A,\overline{\theta}]\\ [1-\widetilde{F}(\frac{1}{2}\theta_A;k_S^*)][2\theta_i\theta_A-\theta_A^2] & \\ \quad -[2\theta_i b_S^*(0)-b_S^*(0)^2] & \text{if } \theta_i\in[\frac{1}{2}b_S^*(0),\frac{1}{2}\theta_A)\\ [1-\widetilde{F}(\frac{1}{2}\theta_A;k_S^*)][2\theta_i\theta_A-\theta_A^2] & \\ -[1-\widetilde{F}(\frac{1}{2}b_S^*(0);k_S^*)][2\theta_i b_S^*(0)-b_S^*(0)^2] & \text{if } \theta_i\in[0,\frac{1}{2}b_S^*(0))\end{cases}. \tag{A10}$$

Since $k_S^*\geq\frac{1}{2}\theta_A$, the indifference condition of the equilibrium becomes $2k_S^*\theta_A-\theta_A^2=2k_S^*b_S^*(0)-b_S^*(0)^2$.

So the equilibrium cut-point k_S^* must satisfy $k_S^*=\frac{\theta_A+\beta(0;k_S^*)}{2}$. Define a new function $H(k)=k-\frac{\theta_A+\beta(0;k)}{2}$. Notice that: $\forall k\geq\theta_A$, we have $k>\frac{\theta_A+\beta(0;k)}{2}$; and $\forall k\leq\frac{1}{2}\theta_A$, we have $k<\frac{\theta_A+\beta(0;k)}{2}$. Thus we know that $H(k)|_{k\geq\theta_A}>0$ and $H(k)|_{k\leq\frac{1}{2}\theta_A}<0$. Given that $H(k)$ is a continuous function with respect to k, an indifference type k^* always exists. Furthermore we must also have: any equilibrium $k_S^*\in(\frac{1}{2}\theta_A,\theta_A)$.

(2) Although an indifferent type always exists, it does not necessarily indicate the existence of the cut-point equilibrium because we need to check the incentive compatibility constraints for other types. Given equation (A10), we know that the payoff gain function $\frac{V_{diff}(\theta_i;k^*)}{F(k^*)}$ is piecewise linear so that we only need to check the IC conditions (i.e., the conditions in Remark 1) at three points: $\theta_i = \frac{1}{2}\beta(0;k^*), \frac{1}{2}\theta_A, 0$.

$\frac{V_{diff}(\theta_i=\frac{1}{2}\theta_A;k^*)}{F(k^*)} = -[\theta_A\beta(0;k^*) - \beta(0;k^*)^2] < 0$,

$\frac{V_{diff}(\theta_i=\frac{1}{2}\beta(0;k^*);k^*)}{F(k^*)} = [1 - \widetilde{F}(\frac{1}{2}\theta_A;k^*)][\beta(0;k^*)\theta_A - \theta_A^2] < 0$.

So we only need to check the IC condition for the type $\theta_i = 0$.

$\frac{V_{diff}(\theta_i=0;k^*)}{F(k^*)} = [1 - \widetilde{F}(\frac{1}{2}\beta(0;k^*);k^*)]\beta(0;k^*)^2 - [1 - \widetilde{F}(\frac{1}{2}\theta_A;k^*)]\theta_A^2$.

Hence, the indifferent type k^* is an equilibrium if and only if

$[F(k^*) - F(\frac{1}{2}\beta(0;k^*))]\beta(0;k^*)^2 \leq [F(k) - F(\frac{1}{2}\theta_A)]\theta_A^2$.

Let's use the notations $x \triangleq \beta(0;k^*), y \triangleq \theta_A$, so that $k^* = \frac{x+y}{2}$. The IC condition then becomes: $F(k^*)(y^2 - x^2) + x^2F(\frac{1}{2}x) \geq F(\frac{1}{2}y)y^2$.

$\because$ $F(\cdot)$ is convex on $[0,\theta_A]$ $\therefore$ $F(k)\frac{y^2-x^2}{y^2} + \frac{x^2}{y^2}F(\frac{1}{2}x) \geq F(k\frac{y^2-x^2}{y^2} + \frac{x^2}{y^2}\frac{1}{2}x) = F(\frac{(y^2-x^2)+xy}{2y})$.

Because $\frac{(y^2-x^2)+xy}{2y} \geq \frac{1}{2}y$, we always have the IC constraint satisfied when $F(\cdot)$ is convex on $[0,\theta_A]$.

(3) We only show non-existence of the symmetric cut-point equilibrium.

(3.1)

In symmetric cut-point equilibrium (if any), recall that we have the IC constraint

$[F(k^*) - F(\frac{1}{2}\beta(0;k^*))]\beta(0;k^*)^2 \leq [F(k) - F(\frac{1}{2}\theta_A)]\theta_A^2$.

First of all, we want to show that as $\overline{\theta} = \theta_A \to +\infty$, the above condition is violated.

Since $\lim\limits_{\overline{\theta}\to+\infty} F(\cdot;\overline{\theta})$ is well defined and any indifferent type $k^* \geq \frac{1}{2}\theta_A$, $\lim\limits_{\overline{\theta}=\theta_A\to+\infty}$ $\beta(0;k^*)$ is well defined and finite because $\lim\limits_{\theta_A\to+\infty} k^* = +\infty$ and $\lim\limits_{k^*\to+\infty} \beta(0;k^*)$ is well defined and finite.

Therefore $\lim\limits_{\overline{\theta}=\theta_A\to+\infty}[F(k^*) - F(\frac{1}{2}\beta(0;k^*))]\beta(0;k^*)^2 > 0$.

(3.2) We know that$[F(k) - F(\frac{1}{2}\theta_A)]\theta_A^2 \leq [1 - F(\frac{1}{2}\theta_A)]\theta_A^2$.

We now verify that $\lim\limits_{\overline{\theta}\to+\infty}[1-F(\frac{1}{2}\overline{\theta})]\overline{\theta}^2 = 0$. Suppose $w = \lim\limits_{\overline{\theta}\to+\infty}[1-F(\frac{1}{2}\overline{\theta};\overline{\theta})]\overline{\theta}^2$, then $E(\theta^2|\overline{\theta}) \geq y^2(1 - F(y|\overline{\theta})) + \int_0^y x^2 dF(x|\overline{\theta})$ for any $y \geq 0$.

Let $y = \frac{1}{2}\overline{\theta}$ and let $\overline{\theta} \to +\infty$, we have $\lim\limits_{\overline{\theta}\to+\infty} E(\theta^2|\overline{\theta}) \geq \frac{1}{4}w + \lim\limits_{\overline{\theta}\to+\infty} E(\theta^2|\overline{\theta})$ which implies that $w = \lim\limits_{\overline{\theta}\to+\infty}[1 - F(\frac{1}{2}\overline{\theta};\overline{\theta})]\overline{\theta}^2 = 0$.

Thus we have

$[F(k^*) - F(\frac{1}{2}\theta_A)]\theta_A^2 \leq [1 - F(\frac{1}{2}\theta_A)]\theta_A^2 \to 0$ (as $\overline{\theta} = \theta_A \to +\infty$).

As a result, for a sufficiently polarized committee, i.e., when $\overline{\theta} = \theta_A$ and they are both large enough, for any indifferent type $k^* \geq \frac{1}{2}\theta_A$, we get

$[F(k^*) - F(\frac{1}{2}\beta(0;k^*))]\beta(0;k^*)^2 > [F(k) - F(\frac{1}{2}\theta_A)]\theta_A^2$, which indicates that no symmetric cut-point equilibrium exists.

(3.3)

Given any sufficiently large $\overline{\theta}$, there must exist a $\delta > 0$ such that for any $\theta_A \in (\overline{\theta} - \delta, \overline{\theta})$,we have

$[F(k^*) - F(\frac{1}{2}\beta(0;k^*))]\beta(0;k^*)^2 > [F(k) - F(\frac{1}{2}\theta_A)]\theta_A^2$.

If this claim is not true, then $\exists$ a series of $\theta_A's$, i.e., $\theta_t \to \overline{\theta}$, but the IC condition is satisfied and k_t is a cut-point equilibrium associated with θ_t.

It is easy to verify that $\lim\limits_{t\to+\infty} k_t$ is an equilibrium, and

$[F(\lim\limits_{t\to+\infty} k_t) - F(\frac{1}{2}\beta(0;\lim\limits_{t\to+\infty} k_t);\lim\limits_{t\to+\infty} k_t)]\beta(0;\lim\limits_{t\to+\infty} k_t)^2 \leq [F(\lim\limits_{t\to+\infty} k_t) - F(\frac{1}{2}\theta_A)]\theta_A^2$, which is a contradiction with the fact that the IC condition is not satisfied when $\overline{\theta} = \theta_A$. Q.E.D.

Proof *of* **Lemma 2**

(1) Part of the construction of the equilibrium cut-point strategy involves $k_B^*(\theta_A) = k_S^*$.

Voter's payoff gain function in the straw poll

$$V_{diff}(\theta_i; k_S^*) = \begin{cases} \sum_{j \geq q} \binom{n-1}{j} F(k_S^*)^{n-1-j}[1 - F(k_S^*)]^j [V(\theta_i, b_2(j+1), j) - V(\theta_i, b_2(j), j)] \\ +\binom{n-1}{q-1} F(k_S^*)^{n-q}[1 - F(k_S^*)]^{q-1}[V(\theta_i, b_2(q), q-1) - V(\theta_i, b_2(q-1), q-1)] \\ +\sum_{j \leq q-2} \binom{n-1}{j} F(k_S^*)^{n-1-j}[1 - F(k_S^*)]^j [V(\theta_i, b_2(j+1), j) - V(\theta_i, b_2(j), j)] \end{cases}, \tag{A11}$$

where

$$b_2(y) \triangleq \beta(y; k_S^*). \tag{A12}$$

As long as $k_S^* \geq \frac{1}{2}\theta_A$, the payoff gain under polling is simplified to

$$V_{diff}(\theta_i; k_S^*) = \begin{cases} \binom{n-1}{q-1} F(k_S^*)^{n-q}[1 - F(k_S^*)]^{q-1}[V(\theta_i, \theta_A, q-1) - V(\theta_i, b_2(q-1), q-1)] \\ +\sum_{j \leq q-2} \binom{n-1}{j} F(k_S^*)^{n-1-j}[1 - F(k_S^*)]^j [V(\theta_i, b_2(j+1), j) - V(\theta_i, b_2(j), j)] \end{cases} \tag{A13}$$

where

$$V(\theta_i, \theta_A, q-1) = \begin{cases} u_v(\theta_A; \theta_i) & \theta_i \geq \frac{1}{2}\theta_A \\ [1 - \widetilde{F}_{n-q,n-q}(\frac{1}{2}\theta_A; k_S^*)]u_v(\theta_A; \theta_i) & \theta_i < \frac{1}{2}\theta_A \end{cases}. \tag{A14}$$

The payoff gain in the binding institution when the initial proposal is the setter's ideal point θ_A is

$$V_{diff}(\theta_i; k_B^*(\theta_A)), b_1 = \theta_A) = \begin{cases} \binom{n-1}{q-1} F(k_S^*)^{n-q}[1 - F(k_S^*)]^{q-1}[u_v(\theta_A; \theta_i) \\ -V(\theta_i, b_2(q-1), q-1)] \\ +\sum_{j \leq q-2} \binom{n-1}{j} F(k_S^*)^{n-1-j}[1 - F(k_S^*)]^j [V(\theta_i, b_2(j+1), j) \\ -V(\theta_i, b_2(j), j)] \end{cases}, \tag{A15}$$

where

$$b_2(y) \triangleq \beta(y; k_B^*(\theta_A)) = \beta(y; k_S^*). \tag{A16}$$

The two payoff gain functions differ only in the first term.

Because $u_v(\theta_A; \theta_i) \leq [1 - \widetilde{F}_{n-q,n-q}(\frac{1}{2}\theta_A; k_S^*)]u_v(\theta_A; \theta_i) < 0$ (when $\theta_i < \frac{1}{2}\theta_A$), we have following property:

(a) $\forall \theta_i \geq \frac{1}{2}\theta_A, V_{diff}(\theta_i, k_B^*(\theta_A), b_1 = \theta_A) = V_{diff}(\theta_i, k_S^*)$;

(b) $V_{diff}(k_B^*(\theta_A), k_B^*(\theta_A), b_1 = \theta_A) = V_{diff}(k_S^*, k_S^*)$; and

(c) $\forall \theta_i < \frac{1}{2}\theta_A, V_{diff}(\theta_i, k_B^*(\theta_A), b_1 = \theta_A) \leq V_{diff}(\theta_i, k_S^*)$.

Based on the above comparison, we know that no voter with any type has an incentive to deviate from the constructed voting strategy $k_B^*(\theta_A) = k_S^*$.

We can then verify that the setter can replicate the same equilibrium stochastic outcome as in the straw poll.

(2) Now let's finish constructing the function $k_B^*(b_1)$ for $b_1 \neq \theta_A$, and the optimal initial proposal b_1^*.

(2.1) For $n > q \geq 2$, let's define $k_B^*(b_1) \triangleq \overline{\theta}$, for $b_1 \neq \theta_A$. It does not hurt for voter i to cast a negative vote if all the other votes are negative. Hence, no voter has an incentive to deviate from the pooling equilibrium strategy.

Because $b_1 = \theta_A$ is the only way that the setter can induce information disclosure, the optimal initial proposal should be $b_1^* = \theta_A$.

(2.2) For $n = q \geq 2$, we write down each voter's payoff gain function if $k_B^*(b_1) = \overline{\theta}$:

$$V_{diff}(\theta_i, k_B^*(b_1) = \overline{\theta}) = V(\theta_i, \beta(1; k_B^*(b_1) = \overline{\theta}), 0) - V(\theta_i, \beta(0; k_B^*(b_1) = \overline{\theta}), 0).$$

Recall that $\beta(1, \overline{\theta})$ is not well defined and depends on the off-the-equilibrium-path belief because it is the revised proposal when setter receives 1 positive vote but knows that all voters reject the experimental proposal for sure. As long as we set the off-the-equilibrium-path belief such that $\beta(1; \overline{\theta}) = \beta(0; \overline{\theta})$, no voter has an incentive to deviate from the pooling equilibrium strategy. As a result, $k_B^*(b_1) = \overline{\theta}$ can always be supported as a continuation equilibrium.

Because $b_1 = \theta_A$ is the only way that the setter can induce information disclosure, the initial period optimal proposal should be $b_1^* = \theta_A$.

(2.3) When $q = 1$, we know that the payoff gain function of a voter is $F(k_B^*(b_1))[u_v(b_1; \theta_i) - V(\theta_i, \beta(0; k_B^*(b_1)), 0)]$, where

$$V(\theta_i, \beta(0; k_B^*(b_1)), 0)$$
$$= \begin{cases} 2\theta_i \beta(0; k_B^*(b_1)) - \beta(0; k_B^*(b_1))^2 & \text{if } \theta_i \geq \frac{1}{2}\beta(0; k_B^*(b_1)) \\ [1 - \widetilde{F}_{n-1,1}(\frac{1}{2}\beta(0; k_B^*(b_1)); k_B^*(b_1))](2\theta_i \beta(0; k_B^*(b_1)) \\ \quad -\beta(0; k_B^*(b_1))^2) & \text{if } \theta_i < \frac{1}{2}\beta(0; k_B^*(b_1)) \end{cases}.$$

(2.3.1) Let's first define $k_B^*(b_1)$ as following, and then make a little revision.

If $b_1 \in [2\overline{\theta} - \beta\left(0; \overline{\theta}\right), +\infty)$, define $k_B^*(b_1) = \overline{\theta}$;

If $b_1 \in [\beta\left(0; \overline{\theta}\right), 2\overline{\theta} - \beta\left(0; \overline{\theta}\right)]$, define $k_B^*(b_1)$ such that $k_B^*(b_1) = \frac{b_1 + \beta(0; k_B^*(b_1))}{2}$;

If $b_1 \in (0, \beta\left(0; \overline{\theta}\right))$, define $k_B^*(b_1)$ such that $b_1 = \beta\left(0; k_B^*(b_1)\right)$ so that the setter's expected payoff is no more than that in the game without any communication, which is also the same as in the case when $k_B^*(b_1) = \overline{\theta}$;

If $b_1 = 0$, define $k_B^*(b_1) = 0$ so that the setter's payoff is 0.

(2.3.2)

Setter's expected welfare under the cut-point k is

$EU_A(k,b_1) = \sum_{j\geq q}\binom{n}{j}F(k)^{n-j}[1-F(k)]^j[2\theta_A b_1 - b_1^2]$
$+\sum_{j\leq q-1}\binom{n}{j}F(k)^{n-j}[1-F(k)]^j[1-\widetilde{F}_{n-j,n-q+1}(\frac{1}{2}\beta(j;k));k)][2\theta_A\beta(j;k)-\beta(j;k)^2]$.

It is continuous in (k,b_1). We can also verify that $\Gamma \triangleq \{(b_1,k) : b_1 \in [\beta\left(0;\overline{\theta}\right), 2\overline{\theta}-\beta\left(0;\overline{\theta}\right)]$ and $k = \frac{b_1+\beta(0;k)}{2}\}$ is a closed and bounded set.

$b_1 \in [\beta\left(0;\overline{\theta}\right), 2\overline{\theta} - \beta\left(0;\overline{\theta}\right)]$ so that b_1 is bounded. $k = \frac{b_1+\beta(0;k)}{2} > \frac{\beta\left(0;\overline{\theta}\right)}{2}$, and $k = \frac{b_1+\beta(0;k)}{2} \leq \overline{\theta}$, so that k is also bounded. Because $k - \frac{b_1+\beta(0;k)}{2}$ is continous in (k,b_1), Γ is a closed set. As a result, Γ is compact and $\underset{(b_1,k)\in\Gamma}{\arg\max} EU_A(k,b_1)$ should not be empty. Suppose $(b_1^0,k^0) \in \underset{(b_1,k)\in\Gamma}{\arg\max} EU_A(k,b_1)$.

Now let's revise $k_B^*(b_1)$ such that $k_B^*(b_1^0) = k^0$ if it did not hold in the original definition. Given this revision, we can verify that the setter's optimal initial proposal should be $b_1^* = b_1^0$.

(3) We now show that if $b_1 = \theta_A$ induces an interior cut-point $k_B^*(\theta_A)$, it will give the setter a strictly higher expected payoff than in the case without any communication. Suppose b' is the policy proposed by the setter in the static game. Setter's expected welfare under the cut-point $k = k_B^*(\theta_A)$ is

$\sum_{j\geq q}\binom{n}{j}F(k)^{n-j}[1-F(k)]^j[2\theta_A b_1 - b_1^2]$
$+\sum_{j\leq q-1}\binom{n}{j}F(k)^{n-j}[1-F(k)]^j[1-\widetilde{F}_{n-j,n-q+1}(\frac{1}{2}\beta(j;k);k)][2\theta_A\beta(j;k)-\beta(j;k)^2]$
$> \sum_{j\geq q}\binom{n}{j}F(k)^{n-j}[1-F(k)]^j[2\theta_A b' - (b')^2]$
$+\sum_{j\leq q-1}\binom{n}{j}F(k)^{n-j}[1-F(k)]^j[1-\widetilde{F}_{n-j,n-q+1}(\frac{1}{2}b';k)][2\theta_A b' - (b')^2]$
$= [1-F_{n,n-q+1}(\frac{1}{2}b')][2\theta_A b' - (b')^2]$,

which is setter's expected payoff in static game.

Proof *of* **Proposition** *3*

(1) We need to show that even the straw poll does not have an informative equilibrium, $b_1 = \theta_A$ can always induce an interior cut-point equilibrium under the

binding institution. The payoff gain

$$V_{diff}(\theta_i; k_B^*(\theta_A), \theta_A) = F(k_B^*(\theta_A))[(2\theta_i\theta_A - \theta_A^2) - V(\theta_i, \beta(0; k_B^*(\theta_A)), 0)], \quad \text{(A17)}$$

where

$$V(\theta_i, b_2(0), 0) = \\ = \begin{cases} (2\theta_i\beta(0; k_B^*(\theta_A)) - \beta(0; k_B^*(\theta_A))^2) & \text{if} \quad \theta_i \geq \frac{1}{2}\beta(0; k_B^*(\theta_A)) \\ [1 - \widetilde{F}(\frac{1}{2}\beta(0; k_B^*(\theta_A)))](2\theta_i\beta(0; k_B^*(\theta_A)) - \beta(0; k_B^*(\theta_A))^2) & \text{if} \quad \theta_i < \frac{1}{2}\beta(0; k_B^*(\theta_A)) \end{cases}.$$

Since $\theta_A \geq \beta(0; k_B^*(\theta_A) \geq [1-\widetilde{F}(\frac{1}{2}\beta(0; k_B^*(\theta_A)); k_B^*(\theta_A))]\beta(0; k_B^*(\theta_A))$, $V_{diff}(\theta_i; k_B^*(\theta_A), \theta_A)$ is a strictly increasing function in θ_i so that we only need to pin down the indifference condition:

$$2k_B^*(\theta_A)\theta_A - \theta_A^2 = 2k_B^*(\theta_A)\beta(0; k_B^*(\theta_A)) - \beta(0; k_B^*(\theta_A))^2, \quad \text{(A18)}$$

i.e., $2k_B^*(\theta_A) = \theta_A + \beta(0; k_B^*(\theta_A))$. Because $2\theta_A > \theta_A + \lim_{k\to\theta_A} \beta(0; k), 0 < \theta_A + \lim_{k\to 0} \beta(0; k)$, and $2k - \theta_A - \beta(0; k)$ is continuous in k, an indifference type $k_B^*(\theta_A)$ always exists so that $b_1 = \theta_A$ always induces an informative sub-form equilibrium. To complete the construction, we can define $k_B^*(b_1)$ when $b_1 \neq \theta_A$ by the same way as in (2.3) in the proof of Lemma 2, so that the optimal initial proposal b_1^* exists.

(2) Whenever the straw poll induces a cut-point equilibrium, it must be the case $k_S^* \geq \frac{1}{2}\theta_A$ so that according to Lemma 2, the binding vote always has an informative equilibrium that makes the setter weakly better than under the straw poll. Q.E.D.

Proof *of* **Proposition 4**

Each voter's (expected) payoff gain is

$$V_{diff}(\theta;k_S^*) = \begin{cases} (1-F(k))[V(\theta_i, b_S^*(2),1) - V(\theta_i, b_S^*(1),1)] \\ +F(k)[V(\theta_i, b_S^*(1),0) - V(\theta_i, b_S^*(0),0)] \end{cases}, \qquad \text{(A19)}$$

where

$$V(\theta_i, b_S^*(2),1) - V(\theta_i, b_S^*(1),1) =$$

$$\begin{cases} [1-\widehat{F}(\tfrac{1}{2}b_S^*(2);k_S^*)][2\theta_i b_S^*(2) - b_S^*(2)^2] & \\ \qquad -[2\theta_i b_S^*(1) - b_S^*(1)^2] & \text{if} \quad \theta_i \geq \tfrac{1}{2}b_S^*(2) \\ \qquad -[2\theta_i b_S^*(1) - b_S^*(1)^2] & \text{if} \quad \tfrac{1}{2}b_S^*(1) \leq \theta_i < \tfrac{1}{2}b_S^*(2) \\ \qquad 0 & \text{if} \quad \theta_i < \tfrac{1}{2}b_S^*(1) \end{cases}$$

$$V(\theta_i, b_S^*(1),0) - V(\theta_i, b_S^*(0),0) =$$

$$\begin{cases} [1-\widetilde{F}(\tfrac{1}{2}b_S^*(1);k_S^*)][2\theta_i b_S^*(1) - b_S^*(1)^2] & \\ -[1-\widetilde{F}(\tfrac{1}{2}b_S^*(0);k_S^*)][2\theta_i b_S^*(0) - b_S^*(0)^2] & \text{if} \quad \theta_i \geq \tfrac{1}{2}b_S^*(1) \\ -[1-\widetilde{F}(\tfrac{1}{2}b_S^*(0);k_S^*)][2\theta_i b_S^*(0) - b_S^*(0)^2] & \text{if} \quad \tfrac{1}{2}b_S^*(0) \leq \theta_i < \tfrac{1}{2}b_S^*(1) \\ \qquad 0 & \text{if} \quad \theta_i < \tfrac{1}{2}b_S^*(0) \end{cases}$$

(1) We first establish the following **claim**:

k_S^* is a cut-point equilibrium if and only if $V_{diff}(\theta_i = k_S^*; k_S^*) = 0$.

This claim is true in the standard signaling game because of the exogenously assumed single-crossing condition on the utility function so that $V_{diff}(\theta_i; k_S^*)$ is a monotonic function in θ_i. However in our game, $V_{diff}(\theta_i; k_S^*)$ is not monotonic in θ_i. We show the above claim in 3 steps.

The First step: to show that $[1-\widehat{F}(\frac{1}{2}\beta(2;k);k)]\beta(2;k) > \beta(1;k)$, so that $V(\theta_i, b_S^*(2),1) - V(\theta_i, b_S^*(1),1)$ is strictly increasing in θ_i when $\theta_i \geq \frac{1}{2}b_S^*(2)$.

As long as $[1-\widehat{F}(\frac{1}{2}\beta(2;k);k)]\beta(2;k) \geq 2k$, we will get $[1-\widehat{F}(\frac{1}{2}\beta(2;k);k)]\beta(2;k) > \beta(1;k)$ because $2k > \beta(1;k)$. Thus we need to show that $[1-\widehat{F}(\frac{1}{2}\beta(2;k);k)]\beta(2;k) \geq 2k$, which is equivalent to $[1-\widehat{F}(\frac{1}{2}\beta(2;k);k)]\beta(2;k) \geq [1-\widehat{F}(\frac{1}{2}\cdot 2k;k)]\cdot 2k$.

If $\arg\max_x [1-\widehat{F}(\frac{1}{2}x;k)]x \geq \beta(2;k)$, $\beta(2;k)$ and $2k$ will be in the interval where $[1-\widehat{F}(\frac{1}{2}x;k)]x$ is strictly increasing (in x).

Thus we will need to show that

$\arg\max_x [1-\widehat{F}(\frac{1}{2}x;k)]x \geq \arg\max_x [1-\widehat{F}(\frac{1}{2}x;k)]^2[2\theta_A x - x^2]$.

Because $\frac{1-\widehat{F}(\frac{1}{2}x;k)}{2\widehat{f}(\frac{1}{2}x;k)} \leq \frac{1-\widehat{F}(\frac{1}{2}x;k)}{\widehat{f}(\frac{1}{2}x;k)}$, we only need to verify that $\frac{2\theta_A x - x^2}{2\theta_A - 2x} \geq x$, which is obviously true. As a result, $\arg\max_x [1-\widehat{F}(\frac{1}{2}x;k)]x \geq \arg\max_x [1-\widehat{F}(\frac{1}{2}x;k)]^2[2\theta_A x - x^2]$.

The Second step: to show that $[1-\widetilde{F}(\frac{1}{2}\beta(1;k);k)]\beta(1;k) \geq [1-\widetilde{F}(\frac{1}{2}\beta(0;k);k)]\beta(0;k)$.

Similarly as in the last step, we can prove that

$\arg\max_x [1-\widetilde{F}(\frac{1}{2}x;k)]x \geq \arg\max_x [1-\widetilde{F}(\frac{1}{2}x;k)][2\theta_A x - x^2]$.

Because $\beta(1;k) \geq \beta(0;k)$ and the function $[1-\widetilde{F}(\frac{1}{2}x;k)][2\theta_A x - x^2]$ is single peaked in x, we have

$[1-\widetilde{F}(\frac{1}{2}\beta(1;k);k)]\beta(1;k) \geq [1-\widetilde{F}(\frac{1}{2}\beta(0;k);k)]\beta(0;k)$.

The Third step:

According to the above two steps, we know that the shapes of the two functions

$\Delta_1(\theta_i) \triangleq V(\theta_i, b_S^*(2), 1) - V(\theta_i, b_S^*(1), 1)$,

$\Delta_0(\theta_i) \triangleq V(\theta_i, b_S^*(1), 0) - V(\theta_i, b_S^*(0), 0)$,

as functions of θ_i, are U-shaped. We draw the two functions in Figure 3.

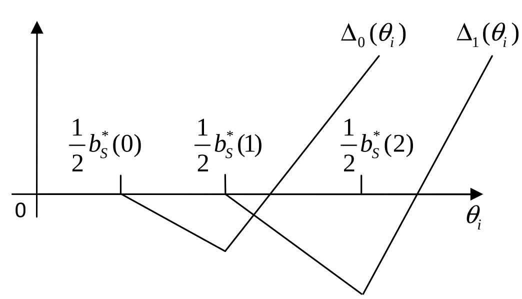

FIGURE 3. DECOMPOSING THE PAYOFF GAINS

The payoff gain function is the weighted average of the above two functions, which can be either the left graph or the right graph in Figure 4. Because of the

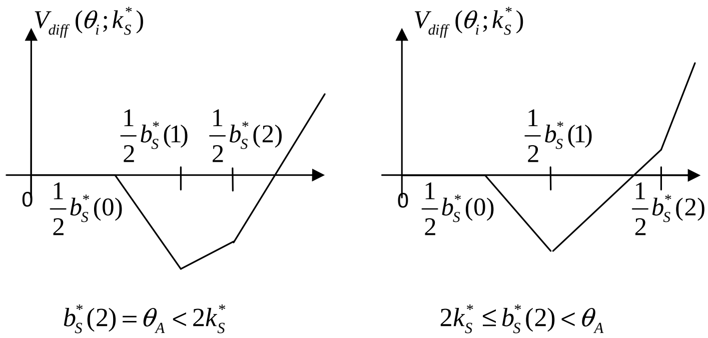

FIGURE 4. PAYOFF GAINS

piecewise linearity of the payoff gain functions, as long as an indifferent type exists, other incentive compatibility constraints are automatically satisfied, although the single-crossing condition is violated.

(2)

(2.1) We will show that when k is sufficiently close to $\overline{\theta}, V_{diff}(k;k) > 0$.

$$\lim_{k\to\overline{\theta}} V_{diff}(k;k)$$
$$= [1-F(\tfrac{1}{2}\beta(1;\overline{\theta}))][2\overline{\theta}\beta(1;\overline{\theta}) - \beta(1;\overline{\theta})^2] - [1-F(\tfrac{1}{2}\beta(0;\overline{\theta}))][2\overline{\theta}\beta(0;\overline{\theta}) - \beta(0;\overline{\theta})^2]$$
$$\geq [1-F(\tfrac{1}{2}\beta(1;\overline{\theta}))][2\theta_A\beta(1;\overline{\theta}) - \beta(1;\overline{\theta})^2] - [1-F(\tfrac{1}{2}\beta(0;\overline{\theta}))][2\theta_A\beta(0;\overline{\theta}) - \beta(0;\overline{\theta})^2]$$
$$> 0$$

The first inequality is due to the fact that

$[1-F(\frac{1}{2}\beta(1;k))]\beta(1;k) > [1-F(\frac{1}{2}\beta(1;k))]\beta(1;k)$.

The second inequality is due to the definition of $\beta(1;k)$.

(2.2) We will show that when k is sufficiently close to $0, V_{diff}(k;k) < 0$.

When k is close to 0,

$$\frac{V_{diff}(k;k)}{2\beta(1;k)k} = \begin{cases} F(k)\{[1-\widetilde{F}(\frac{1}{2}\beta(1;k);k)][1-\frac{\beta(1;k)^2}{2\beta(1;k)k}] - [1-\widetilde{F}(\frac{1}{2}\beta(0;k);k)][\frac{\beta(0;k)}{\beta(1;k)} - \frac{\beta(0;k)}{\beta(1;k)}\frac{\beta(0;k)^2}{2k\beta(0;k)}]\} \\ +(1-F(k))[-1+\frac{\beta(1;k)}{2k}] \end{cases} \tag{A20}$$

Because$\frac{\beta(1;k)^2}{2\beta(1;k)k}, \frac{\beta(0;k)}{\beta(1;k)}, \frac{\beta(0;k)^2}{2k\beta(0;k)}$ are bounded, $\lim_{k\to 0}\frac{V_{diff}(k;k)}{2\beta(1;k)k} = \lim_{k\to 0}\frac{\beta(1;k)}{2k} - 1$.

Recall that $\beta(1;k)$ is pinned down by $\frac{F(k)-F(\frac{1}{2}b)}{f(\frac{1}{2}b)} = \frac{1}{2}\frac{2\theta_A b-b^2}{2(\theta_A-b)}$. This first order condition implies a monotonic relationship between k and $b=\beta(1;k)$. When $k\to 0^+$, since $\beta(1;k)\in(0,2k)$, we have $b=\beta(1;k)\to 0^+$.

$$\begin{aligned}\lim_{k\to 0^+}\frac{\beta(1;k)}{2k} &= \lim_{b\to 0^+}\frac{b}{2F^{-1}[\frac{1}{2}\frac{2\theta_A b-b^2}{2(\theta_A-b)}f(\frac{1}{2}b)+F(\frac{1}{2}b)]}\\ &= \frac{1}{2}\lim_{b\to 0}\frac{f(0)}{[(\frac{1}{2}\frac{2\theta_A b-b^2}{2(\theta_A-b)})'f(\frac{1}{2}b)+(\frac{1}{2}\frac{2\theta_A b-b^2}{2(\theta_A-b)})f'(\frac{1}{2}b)\frac{1}{2}+f(\frac{1}{2}b)\frac{1}{2}]}\\ &= \frac{1}{2}\frac{f(0)}{\frac{1}{2}f(0)+f(0)\frac{1}{2}}\\ &= \frac{1}{2}\end{aligned}$$

The first equation above comes from $\frac{F(k)-F(\frac{1}{2}b)}{f(\frac{1}{2}b)} = \frac{1}{2}\frac{2\theta_A b-b^2}{2(\theta_A-b)}$, so that $F(k) = \frac{1}{2}\frac{2\theta_A b-b^2}{2(\theta_A-b)}f(\frac{1}{2}b)+F(\frac{1}{2}b)$.

As a result, $\lim_{k\to 0}\frac{b(1,k)}{2k} < 1$ and $\lim_{k\to 0}\frac{V_{diff}(k;k)}{2b_2(1;k)k} < 0$.

Thus $\exists k_0 > 0$ such that $V_{diff}(k_0;k_0) < 0$.

(2.3) Because $V_{diff}(k;k)$ is a continuous function in k, $\exists k_S^* \in (0,\overline{\theta})$ such that $V_{diff}(k_S^*;k_S^*) = 0$. Q.E.D.

Proof *of* **Lemma** 3

(1) With 2 voters and the unanimity rule, suppose there is an equilibrium cut-point $k_S^* \le \frac{1}{2}\theta_A$. We have the indifference condition:

$$-[1-F(k_S^*)][2k_S^* b_S^*(1) - b_S^*(1)^2] + [F(k_S^*) - F(\tfrac{1}{2}b(1))][2k_S^* b_S^*(1) - b_S^*(1)^2]$$
$$-[F(k_S^*) - F(\tfrac{1}{2}b(0))][2k_S^* b_S^*(0) - b_S^*(0)^2] = 0,$$

i.e.,

$$[2F(k_S^*)-F(\frac{1}{2}b_S^*(1))-1][2k_S^* b_S^*(1)-b_S^*(1)^2] = [F(k_S^*)-F(\frac{1}{2}b_S^*(0))][2k_S^* b_S^*(0)-b_S^*(0)^2]. \quad \text{(A21)}$$

Since the left-hand side should be positive, we must have

$$2F(k_S^*) > F(\frac{1}{2}b_S^*(1)) + 1. \quad \text{(A22)}$$

(1.1) Whenever $F(\frac{1}{2}\theta_A) \leq \frac{1}{2}$ or $F(\frac{1}{2}\overline{\theta}) \leq \frac{1}{2}$, we get $2F(k_S^*) \leq 2F(\frac{1}{2}\theta_A) \leq 1$, so that $2F(k_S^*) - F(\frac{1}{2}b_S^*(1)) < 1$ which contradicts with inequality (A22).

As a result, whenever $\theta_A \leq 2F^{-1}(\frac{1}{2})$ or $\overline{\theta} \leq 2F^{-1}(\frac{1}{2})$, any equilibrium cut-point k_S^* must be strictly greater than$\frac{1}{2}\theta_A$.

(1.2) When F is convex, inequality (A22) implies

$F(k_S^*) > \frac{1}{2}F(\frac{1}{2}b_S^*(1)) + \frac{1}{2} \cdot 1 \geq F(\frac{1}{2} \cdot \frac{1}{2}b_S^*(1) + \frac{1}{2}\overline{\theta})$.

We then get $k_S^* > \frac{1}{2} \cdot \frac{1}{2}b_S^*(1) + \frac{1}{2}\overline{\theta}$, therefore $\frac{1}{2}\theta_A > \frac{1}{2} \cdot \frac{1}{2}b_S^*(1) + \frac{1}{2}\overline{\theta}$ so that $\theta_A > \overline{\theta}$.This is a contradiction.

As a result, when $F(\cdot)$ is convex, any equilibrium symmetric cut-point $k_S^* > \frac{1}{2}\theta_A$.

(2)

(2.1) First we show that $\exists M_0 > 0$ such that for all $\theta_A = \overline{\theta} \geq M_0$, $k_S^* < \frac{1}{2}\theta_A$. If it is not true, $\exists$ a series of setter's ideal points (indexed by t), $\theta_t \to +\infty$ ($\theta_A = \overline{\theta} = \theta_t$) and the associated equilibrium $k_t \geq \frac{1}{2}\theta_t$.

Recall that $V_{diff}(k;k) = (1 - F(k))[2k\theta_A - \theta_A^2 - (2k\beta(1;k) - \beta(1;k)^2)]$

$+[F(k) - F(\frac{1}{2}\beta(1;k))][2k\beta(1;k) - \beta(1;k)^2] - [F(k) - F(\frac{1}{2}\beta(0;k))][2k\beta(0;k) - \beta(0;k)^2]$.

We can verify that

(2.1.a) $\lim\limits_{t\to+\infty} \beta(1;\theta_t)$ and $\lim\limits_{t\to+\infty} \beta(0;\theta_t)$ finitely exist (according to the first order conditions of setter's optimization), and $\lim\limits_{t\to+\infty} \beta(1;\theta_t) > \lim\limits_{t\to+\infty} \beta(0;\theta_t)$.

Specifically, $\lim\limits_{t\to+\infty} \beta(1;\theta_t)$ is the equilibrium proposal when the setter (with utility b) faces one voter in the static game; and

$\lim\limits_{t\to+\infty} \beta(0;\theta_t)$ is the equilibrium proposal when the setter (with utility b) faces two voters(with the unanimity rule) in the static game.

(2.1.b)

$$\lim_{t\to+\infty}\frac{V_{diff}(k_t;k_t)}{2k_t}=\begin{cases}\lim_{t\to+\infty}\{(1-F(k_t))(\theta_t-\beta(1;k_t))[1-\frac{\theta_t}{2k_t}] \\ -(1-F(k_t))(\frac{\theta_t}{2k_t}-\frac{\beta(1;k_t)}{2k_t})\beta(1;k_t)\} \\ +\lim_{t\to+\infty}\{[F(k_t)-F(\frac{1}{2}\beta(1;k_t))][\beta(1;k_t)-\frac{\beta(1;k_t)^2}{2k_t}] \\ -[F(k_t)-F(\frac{1}{2}\beta(0;k_t))][\beta(0;k_t)-\frac{\beta(0;k_t)^2}{2k_t}]\}\end{cases}$$

$$\geq \lim_{t\to+\infty}\{[F(k_t)-F(\tfrac{1}{2}\beta(1;k_t))]\beta(1;k_t)-[F(k_t)-F(\tfrac{1}{2}\beta(0;k_t))]\beta(0;k_t)\}$$

$$>0$$

The last inequality comes from the fact that $\lim_{t\to+\infty}[F(k_t)-F(\frac{1}{2}\beta(1;k_t))]\beta(1;k_t)$ is setter (with preference b)'s optimal expected payoff when she faces only one voter without any communication, which must be strictly greater than the payoff if she chooses a different proposal $\lim_{t\to+\infty}\beta(0;k_t)$. The inequality suggests that for a large enough t, the indifference condition as a local incentive compatibility condition is violated. This is a contradiction. As a result, $\exists M_0>0$ such that for all $\theta_A=\overline{\theta}\geq M_0$, $k_S^*<\frac{1}{2}\theta_A$.

(2.2)We then claim that: given any $\overline{\theta}\geq M_0$, for θ_A sufficiently close to $\overline{\theta}$, any equilibrium cut-point $k_S^*<\frac{1}{2}\theta_A$.

If the claim is not true, we then have $\theta_A=\theta_t\to\overline{\theta}, k_t^*\geq\frac{1}{2}\theta_A$

$\because V_{diff}(k_t;k_t)=0, V_{diff}(\theta;k_t)\geq 0$ (for $\theta>k_t$), $V_{diff}(\theta;k_t)\leq 0$ (for $\theta<k_t$)

$\therefore V_{diff}(\lim_{t\to+\infty}k_t;\lim_{t\to+\infty}k_t)=\lim_{t\to+\infty}V_{diff}(k_t;k_t)=0$,

$V_{diff}(\theta;\lim_{t\to+\infty}k_t)\geq 0$ (for $\theta>\lim_{t\to+\infty}k_t$), and $V_{diff}(\theta;\lim_{t\to+\infty}k_t)\leq 0$ (for $\theta<\lim_{t\to+\infty}k_t$).[28]

$\therefore \lim_{t\to+\infty}k_t\geq\frac{1}{2}\theta_A$ is a cut-point equilibrium when $\theta_A=\overline{\theta}$.[29]

This claim contradicts the claim in (2.1) that when $\theta_A=\overline{\theta}$ is sufficiently large,

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[28]In detail, for any fixed $\theta>\lim_{t\to+\infty}k_t$, as long as t is large enough, we have $\theta>k_t$, so that $V_{diff}(\theta;k_t)\geq 0$,therefore $V_{diff}(\theta;\lim_{t\to+\infty}k_t)=\lim_{t\to+\infty}V_{diff}(\theta;k_t)\geq 0$. A similar proof applies to the other situation.

[29]One can verify that $\lim_{t\to+\infty}k_t<\overline{\theta}$, otherwise we have $V_{diff}(\lim_{t\to+\infty}k_t;\lim_{t\to+\infty}k_t)=[1-F(\frac{1}{2}\beta(1;k_t))][2\theta_A\beta(1;k_t)-\beta(1;k_t)^2]-[1-F(\frac{1}{2}\beta(0;k_t))][2\theta_A\beta(0;k_t)-\beta(0;k_t)^2]>0$.

any equilibrium $k_S^* < \frac{1}{2}\theta_A$. Thus, given any $\overline{\theta} \geq M_0$, for θ_A sufficiently close to $\overline{\theta}$, we have $k_S^* < \frac{1}{2}\theta_A$. Q.E.D.

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Supplementary Appendix

This appendix supplements *Communication in Collective Bargaining* and will be made available on the web. Here we provide detailed proofs for some technical results and extensions of the model. We do not cover the results that are proven directly in the paper or Appendix attached with the paper.

Characterizing the Second Period Proposal

Lemma 6 *(Basic Properties of Order Statistics) Suppose $F_{x,y}$ is the distribution function representing the y th smallest random variable among the x i.i.d. random variables with distribution $F(\cdot)$ and probability density function $f(\cdot), (x, y \in \mathbb{Z}^+, x \geq y)$ then we have:*

(1) $\frac{1-F_{x,y}(\theta)}{f_{x,y}(\theta)} = \sum_{i=0}^{y-1} \frac{(y-1)!(x-y)!}{(x-i)!(i)!} (\frac{1-F(\theta)}{F(\theta)})^{y-i-1} \frac{1-F(\theta)}{f(\theta)}$;

(2) $\frac{1-F_{x,y}(\theta)}{f_{x,y}(\theta)}$ *is decreasing in θ provided $F(\cdot)$ satisfies increasing hazard rate property; when $y \geq 2$,* $\frac{1-F_{x,y}(\theta)}{f_{x,y}(\theta)}$ *is strictly decreasing in θ*

(3) $\frac{1-F_{x,y}(\theta)}{f_{x,y}(\theta)} > \frac{1-F_{x+1,y}(\theta)}{f_{x+1,y}(\theta)}$;

(4) $\frac{1-F_{x,y}(\theta)}{f_{x,y}(\theta)} < \frac{1-F_{x,y+1}(\theta)}{f_{x,y+1}(\theta)}$ *and* $\frac{1-F_{x,y}(\theta)}{f_{x,y}(\theta)} < \frac{1-F_{x+1,y+1}(\theta)}{f_{x+1,y+1}(\theta)}$.

Proof *of* **Lemma 6**

(1) Generally the distribution function $F_{x,y}(\theta)$ and the probability density function $f_{x,y}(\theta)$ of the y th smallest order statistics from x i.i.d. random variables are given by:

$$1 - F_{x,y}(\theta) = \sum_{i=0}^{y-1} \binom{x}{i} F^i (1-F)^{x-i} \tag{S1}$$

$$f_{x,y}(\theta) = \frac{x!}{(y-1)!(x-y)!} F^{y-1} (1-F)^{x-y} f \tag{S2}$$

By calculation, we get

$$\frac{1-F_{x,y}(\theta)}{f_{x,y}(\theta)} = \sum_{i=0}^{y-1} \frac{(y-1)!(x-y)!}{(x-i)!(i)!} (\frac{1-F(\theta)}{F(\theta)})^{y-i-1} \frac{1-F(\theta)}{f(\theta)} \tag{S3}$$

(2) From the expression above we simply know that $\frac{1-F_{x,y}(\theta)}{f_{x,y}(\theta)}$ is strictly decreasing in θ provided $F(\cdot)$ satisfies increasing hazard rate property and $y \geq 2$; When $y = 1$,

$$\frac{1-F_{x,1}(\theta)}{f_{x,1}(\theta)} = \frac{1}{x} \frac{1-F(\theta)}{f(\theta)} \tag{S4}$$

which is weakly decreasing in θ.

(3) To show $\frac{1-F_{x,y}(\theta)}{f_{x,y}(\theta)} > \frac{1-F_{x+1,y}(\theta)}{f_{x+1,y}(\theta)}$ is decreasing in x, we have

$\frac{1-F_{x,y}(\theta)}{f_{x,y}(\theta)} > \frac{1-F_{x+1,y}(\theta)}{f_{x+1,y}(\theta)}$

$\Leftrightarrow \sum_{i=0}^{y-1} \frac{(y-1)!(x+1-y)!}{(x+1-i)!(i)!} (\frac{1-F(\theta)}{F(\theta)})^{y-i-1} \frac{1-F(\theta)}{f(\theta)} < \sum_{i=0}^{y-1} \frac{(y-1)!(x-y)!}{(x-i)!(i)!} (\frac{1-F(\theta)}{F(\theta)})^{y-i-1} \frac{1-F(\theta)}{f(\theta)}$

$\Leftrightarrow \sum_{i=0}^{y-1} \frac{(x+1-y)}{(x+1-i)(x-i)!(i)!} (\frac{1-F(\theta)}{F(\theta)})^{y-i-1} < \sum_{i=0}^{y-1} \frac{1}{(x-i)!(i)!} (\frac{1-F(\theta)}{F(\theta)})^{y-i-1}$

$\Leftarrow \frac{(x+1-y)}{(x+1-i)(x-i)!(i)!} < \frac{1}{(x-i)!(i)!} \forall 0 \leqslant i \leqslant y-1$

$\Leftrightarrow x+1-y < x+1-i, \forall 0 \leqslant i \leqslant y-1$

$\Leftrightarrow i < y, \forall 0 \leqslant i \leqslant y-1$

(4) To show $\frac{1-F_{x,y}(\theta)}{f_{x,y}(\theta)}$ is increasing in y, we only need to show $\frac{1-F_{x,y}(\theta)}{f_{x,y}(\theta)} < \frac{1-F_{x+1,y+1}(\theta)}{f_{x+1,y+1}(\theta)}$ because of following fact:

$\frac{1-F_{x,y}(\theta)}{f_{x,y}(\theta)} < \frac{1-F_{x+1,y+1}(\theta)}{f_{x+1,y+1}(\theta)} < \frac{1-F_{x,y+1}(\theta)}{f_{x,y+1}(\theta)}$.

We have

$\frac{1-F_{x+1,y+1}(\theta)}{f_{x+1,y+1}(\theta)} = \sum_{i=0}^{y} \frac{y!(x-y)!}{(x+1-i)!(i)!} (\frac{1-F(\theta)}{F(\theta)})^{y-i} \frac{1-F(\theta)}{f(\theta)}$

$= \sum_{i=1}^{y} \frac{y!(x-y)!}{(x+1-i)!(i)!} (\frac{1-F(\theta)}{F(\theta)})^{y-i} \frac{1-F(\theta)}{f(\theta)} + \frac{y!(x-y)!}{(x+1)!} (\frac{1-F(\theta)}{F(\theta)})^{y} \frac{1-F(\theta)}{f(\theta)}$

$> \sum_{i=1}^{y} \frac{y!(x-y)!}{(x+1-i)!(i)!} (\frac{1-F(\theta)}{F(\theta)})^{y-i} \frac{1-F(\theta)}{f(\theta)}$

$= \sum_{i=0}^{y-1} \frac{(y-1)!(x-y)!}{(x-i)!(i)!} (\frac{1-F(\theta)}{F(\theta)})^{y-i-1} \frac{1-F(\theta)}{f(\theta)} \frac{y}{(i+1)}$

$\geq \frac{1-F_{x,y}(\theta)}{f_{x,y}(\theta)}$

Q.E.D.

We use Lemma 6 to show the following lemma.

Lemma 7 *Suppose* $\widetilde{F}(x) \triangleq \frac{F(x)}{F(k)}$ *when* $x \in [0,k]$, $\widehat{F}(x) \triangleq \frac{F(x)-F(k)}{1-F(k)}$ *when* $x \in [k,\overline{\theta}]$. $\widetilde{F}_{n-y,n-q+1}$ *is the (n-q+1)th smallest order statistics among the* $(n-y)$*(with* $y \leq q-1$*) i.i.d. random variables with distribution* $\widetilde{F}(x)$. $\widehat{F}_{y,y-q+1}$ *is the* $(y-q+1)$*th smallest order statistics among the* y *(with* $y \geq q$*) i.i.d. random variables with distribution* $\widehat{F}(x)$*. The following statements are true under Assumption 1*

(1) $\frac{1-\widetilde{F}_{n-y,n-q+1}(\theta;k)}{\widetilde{f}_{n-y,n-q+1}(\theta;k)}$ *is strictly increasing in* y*, strictly decreasing in* θ*, strictly increasing and continuously differentiable in* k*;*

(2) $\frac{1-\widehat{F}_{y,y-q+1}(\theta;k)}{\widehat{f}_{y,y-q+1}(\theta;k)}$ *is strictly increasing in* y*,decreasing in* θ[1]*, is independent of* k *whenever* $y=q$*.*

Proof *of* **Lemma 7**

(1) According to Lemma 6, $\frac{1-\widetilde{F}_{n-y,n-q+1}(\theta;k)}{\widetilde{f}_{n-y,n-q+1}(\theta;k)}$ is strictly strictly increasing in y. Given the detailed expression of $\frac{1-\widetilde{F}_{n-y,n-q+1}(\theta;k)}{\widetilde{f}_{n-y,n-q+1}(\theta;k)}$,

$$\frac{1-\widetilde{F}_{n-y,n+1-q}(\theta;k)}{\widetilde{f}_{n-y,n+1-q}(\theta;k)} = \sum_{i=0}^{n-q} \frac{(n-q)![n-y-(n+1-q)]!}{(n-y-i)!(i)!} (\frac{F(k)-F(\theta)}{F(\theta)})^{n-q-i} \frac{F(k)-F(\theta)}{f(\theta)} \tag{S5}$$

we find that it is strictly increasing and continuously differentiable in k. In the following we show that

$\frac{A-F(x)}{f(x)}$ is strictly decreasing for $F(x) < A, 0 < A < 1$, so that $\frac{1-\widetilde{F}_{n-y,n+1-q}(\theta;k)}{\widetilde{f}_{n-y,n+1-q}(\theta;k)}$is (strictly) decreasing in θ.

$(\frac{1-F}{f})' = \frac{-f^2-(1-F)f'}{f^2} \leq 0 \Rightarrow f' \geqslant -\frac{f^2}{1-F}$

$0 < A < 1 \Rightarrow 0 < A-F < 1-F \Rightarrow -\frac{1}{1-F} > -\frac{1}{A-F}$

thus $f' \geqslant -\frac{f^2}{1-F} > -\frac{f^2}{A-F}$.

[1] It is strictly decreasing in θ whenever $y > q$ or $\frac{1-F(\theta)}{f(\theta)}$ is strictly decreasing.

As a result $(\frac{A-F}{f})' = \frac{-f^2-(A-F)f'}{f^2} < 0$. Therefore $\frac{1-\widetilde{F}_{n-y,n+1-q}(\theta;k)}{\widetilde{f}_{n-y,n+1-q}(\theta;k)}$is (strictly) decreasing in θ.

(2) Similar as (1), we have following property directly from Lemma 6**:** $\frac{1-\widehat{F}_{y,y-q+1}(\theta;k)}{\widehat{f}_{y,y-q+1}(\theta;k)}$is strictly decreasing in r. Notice that $\frac{1-\widehat{F}(x)}{\widehat{f}(x)} = \frac{1-F(x)}{f(k)}$(When $x > k$). We write down the detailed expression of $\frac{1-\widehat{F}_{y,y-q+1}(\theta;k)}{\widehat{f}_{y,y-q+1}(\theta;k)}$

$$\frac{1-\widehat{F}_{y,y-q+1}(\theta;k)}{\widehat{f}_{y,y-q+1}(\theta;k)} = \sum_{i=0}^{y-q}\frac{(y-q)!(q-1)!}{(y-i)!(i)!}(\frac{1-F(\theta)}{F(\theta)})^{y-q-i}\frac{1-F(\theta)}{f(\theta)} \tag{S6}$$

It implies that $\frac{1-\widehat{F}_{y,y-q+1}(\theta;k)}{\widehat{f}_{y,y-q+1}(\theta;k)}$is decreasing in θ,and is independent of k whenever $y = q$. Q.E.D.

Proof *of* **Lemma 4**

For convenience, we use Eu_A to represent setter's expected utility at the beginning of the second period.

(1) The proof consists of three steps. In the first step, we rule out the possibility that optimal $\frac{1}{2}b$ is outside of the support of the distribution. Based on this we can then take derivative and argue that the objective function of the setter is inverse u-shaped (i.e. single peaked). Third, we use the sign of derivatives at the boundary to pin down whether optimal proposal is a corner solution or not.

The *first step*

We observe that $\forall b \in (0, 2\theta_A) \cap (-\infty, 2k)$, we have $0 < \frac{1}{2}b < \theta_A$ and $\frac{1}{2}b < k$, thus $Eu_A \mid_b > 0$, which implies that $\forall b \in [2k, +\infty) \cup \{0\}$ (gives the setter non-positive expected payoff) is strictly dominated by $b \in (0, 2\theta_A) \cap (-\infty, 2k)$.

If $2k \leq \theta_A$, we have "optimal" $b < 2k \leq \theta_A$.

If $\theta_A < 2k, \forall b \in (\theta_A, 2k]$ is strictly dominated by $b - \epsilon$ for some small positive ϵ, therefore $\widetilde{b}(y) \leq \theta_A < 2k$.

As a summary of above argument, we have $\widetilde{b}(y) \in (0, 2k)$ and $\widetilde{b}(y) \leq \theta_A$.

The second step

By Lemma 7 and the following derivative, we know that $Eu_A = [1 - \widetilde{F}_{n-y,n-q+1}(\frac{1}{2}b)]u_A(b)$ is single peaked because $\frac{1-\widetilde{F}_{n-y,n-q+1}(\frac{1}{2}b)}{\widetilde{f}_{n-y,n-q+1}(\frac{1}{2}b)}$ is decreasing and $\frac{1}{2}\frac{u_A(b)}{u'_A(b)}$ is increasing.

$\frac{dEu_A}{db} = [1 - \widetilde{F}_{n-y,n-q+1}(\frac{1}{2}b)]u'_A(b) - \widetilde{f}_{n-y,n-q+1}(\frac{1}{2}b)\frac{1}{2}u_A(b)$

$= u'_A(b)\widetilde{f}_{n-y,n-q+1}(\frac{1}{2}b)[\frac{1-\widetilde{F}_{n-y,n-q+1}(\frac{1}{2}b)}{\widetilde{f}_{n-y,n-q+1}(\frac{1}{2}b)} - \frac{1}{2}\frac{u_A(b)}{u'_A(b)}]$

The third step

If $\theta_A < 2k$, $\frac{dEu_A}{db}\mid_{b=\theta_A} = -\widetilde{f}_{n-y,n-q+1}(\frac{1}{2}\theta_A)\frac{1}{2}u_A(\theta_A) < 0$.

As a result, we have $\widetilde{b}(y) \in (0, \min\{2k, \theta_A\})$ and is uniquely determined by the F.O.C.

(2.1) If $0 < \frac{1}{2}\theta_A \leq k < \overline{\theta}$, for any $\theta_i \geq k$

$2\theta_i\theta_A - \theta_A^2 \geq 2k\theta_A - \theta_A^2 \geq 0$

so that he (i.e. the voter with ideal point θ_i) weakly prefers setter's ideal point to the status quo. When $y \geq q$, the "pivotal" ideal point must be above the cut-point, therefore $\widehat{b}(y) = \theta_A$.

(2.2) When $0 < k < \frac{1}{2}\theta_A$, we do the proof in similar steps as in (1).

$Eu_A = [1 - \widehat{F}_{y,y-q+1}(\frac{1}{2}b)]u_A(b)$

First we observe that $b \geq 2\overline{\theta}$ will be strictly dominated by $b \in (0, 2\theta_A)$. Furthermore $b \in (\theta_A, 2\overline{\theta})$ will be strictly dominated by $b - \epsilon$, $\epsilon > 0$ and is very close to 0. In addition, $b < 2k$ is strictly dominated by $b = 2k$.As a summary we have $\widehat{b}(y) \in [2k, \theta_A]$.

Because

$\frac{dEu_A}{db}\mid_{b=\theta_A} = -\widehat{f}_{y,y-q+1}(\frac{1}{2}\theta_A)\frac{1}{2}u_A(\theta_A) < 0,$

we have $\widetilde{b}(y) \in [2k, \theta_A)$.

$\frac{dEu_A}{db}\mid_{b=2k} = u'_A(2k) - \frac{1}{2}\widehat{f}_{y,y-q+1}(k)u_A(2k)$

$\because 0 < 2k < \theta_A \therefore u'_A(2k) > 0$

$\because \widehat{f}_{y,y-q+1}(k) = \begin{cases} 0 & \text{if} \quad q < y \\ q\frac{f(k)}{1-F(k)} & \text{if} \quad q = y \end{cases}$

$\therefore$Only when $y = q$, $\frac{1}{q}\frac{1-F(k)}{f(k)} \leq \frac{1}{2}\frac{u_A(2k)}{u'_A(2k)}$, $\widehat{b}(y) = 2k$, otherwise $\widehat{b}(y) \in (2k, \theta_A)$ and is uniquely determined by the F.O.C. $\frac{1-F_{y,y-q+1}(\frac{1}{2}b;k)}{F_{y,y-q+1}(\frac{1}{2}b;k)} = \frac{1}{2}\frac{u_A(b)}{u'_A(b)}$.

Q.E.D.

Cheap-talk Equilibrium under Simple Majority Rule

Example 1 *Suppose* $\theta_i \tilde{} U(0, \overline{\theta}), n = 2, q = 1, 0 < \theta_A \leq \overline{\theta}$. *We can verify that the cut-point under simple majority is uniquely determined by* $k^* = \frac{1}{2}b_2(2, k^*) + \frac{\theta_A}{2}$.

Proof *of* **Example 1**

According to Lemma 4, $b_2(0; k)$ under simple majority is determined by

$\frac{4k^2-b^2}{b} = \frac{2\theta_A b-b^2}{\theta_A - b}$

Combining it with the cut-point condition i.e. $k = \frac{1}{2}b_2(0; k) + \frac{\theta_A}{2}$we have

$\frac{4k^2-(2k-\theta_A)^2}{(2k-\theta_A)} = (2k - \theta_A)\frac{2\theta_A-(2k-\theta_A)}{\theta_A-(2k-\theta_A)}$

$\Leftrightarrow k^3 - \frac{7}{2}\theta_A k^2 + 3\theta_A^2 k - \frac{5}{8}\theta_A^3 = 0$

Define $\varphi(x) \triangleq x^3 - \frac{7}{2}\theta_A x^2 + 3\theta_A^2 x - \frac{5}{8}\theta_A^3, x \in (\frac{\theta_A}{2}, \theta_A)$

$\varphi'(x) = 3x^2 - 7\theta_A x + 3\theta_A^2, x \in (\frac{\theta_A}{2}, \theta_A)$

$\varphi'(\frac{\theta_A}{2}) = \frac{1}{4}\theta_A^2 > 0, \varphi'(\theta_A) = -\theta_A^2 < 0$

$\varphi(\frac{\theta_A}{2}) = \frac{1}{8}\theta_A^3 > 0, \varphi(\theta_A) = -\frac{1}{8}\theta_A^3 < 0.$

As a result, we know that $\varphi(x)$ is strictly increasing from $\frac{\theta_A}{2}$ to some $\widehat{x} < \theta_A$ and is strictly decreasing from $\widehat{x}$ to θ_A, so that $\varphi(x)$ has a unique root, which is on $(\widehat{x}, \theta_A)$. Q.E.D.

Unanimity Rule

In the straw poll with unanimity rule, as long as the setter is sufficiently moderate, or the committee is sufficiently homogeneous, the information structure is such that $k^* \geq \frac{1}{2}\theta_A$. However, for a sufficiently polarized committee and a polarized setter, $k^* < \frac{1}{2}\theta_A$. This contrasts with the pattern in models with a single veto player (Matthews, 1989).

Similarly as in Lemma 1 with simple majority, Lemma 4 in the Appendix implies that whether k^* is greater than $\frac{1}{2}\theta_A$ determines whether it is possible that the setter proposes her ideal point in the second period. Specifically,

if $k^* < \frac{\theta_A}{2}, \theta_A > \widehat{b}_2(2) \geq 2k > \widetilde{b}_2(1) > \widetilde{b}_2(0)$,

if $k^* \geq \frac{\theta_A}{2}, \theta_A = \widehat{b}_2(2) > 2k > \widetilde{b}_2(1) > \widetilde{b}_2(0)$.

When the committee is sufficiently divergent, according to Lemma 3, $k^* < \frac{\theta_A}{2}$, so that she never proposes her ideal policy and always compromises. When the setter is sufficiently moderate, we have $k^* \geq \frac{\theta_A}{2}$, hence it is possible that she does not compromise. We summarize this property in the following corollary.

Corollary 1 *In a straw poll with unanimity rule,*

(1) as long as one of the following conditions holds, the ex ante probability that the setter makes a compromise proposal is smaller than 1:

[1.1] $\theta_A \leq 2F^{-1}(\frac{1}{2})$ *or* $\overline{\theta} \leq 2F^{-1}(\frac{1}{2})$*; [1.2]* $F(\cdot)$ *is convex; and*

(2) for sufficiently large $\overline{\theta}$, $\exists M_2 < \overline{\theta}$, *s.t. whenever* $\theta_A \in (M_2, \overline{\theta})$*, the setter always compromises and never proposes her ideal point (provided* $\lim_{\overline{\theta} \to +\infty} F(\theta; \overline{\theta})$ *exists).*

In the following we discuss the situation when $k^* < \frac{\theta_A}{2}$ happens in equilibrium. In this case, although the replication result may still hold, the setter can only use some compromising proposal instead of her ideal point. As implied by Lemma 3, the cut-points in the straw poll are smaller than the sincere cut-point when there is a strong preference divergence between the setter and voters. If we want the binding vote can somehow replicate the straw poll outcome, the induced cut-points in the binding institution must be the same as in polling. Furthermore we also need b_1 to coincide with setter's revised proposal when she receives 2 endorsements. According to the proof of Proposition 4, the cut-point equilibrium in polling with unanimity rule and "polarized" committee is determined by

$$(1 - F(k))[0 - (2kb_2(1) - b_2(1)^2)]$$
$$+[F(k) - F(\tfrac{1}{2}b(1))][2kb_2(1) - b_2(1)^2] - [F(k) - F(\tfrac{1}{2}b(0))][2kb_2(0) - b_2(0)^2] = 0$$

Comparing it with the cut-point equation in the binding institution,

$$(1 - F(k))[(2kb_1 - b_1^2) - (2kb_2(1) - b_2(1)^2)]$$
$$+[F(k) - F(\tfrac{1}{2}b(1))][2kb_2(1) - b_2(1)^2] - [F(k) - F(\tfrac{1}{2}b(0))][2kb_2(0) - b_2(0)^2] = 0$$

we know that b_1 can induce the same stochastic outcome in polling only if $2kb_1 - b_1^2 = 0$ (or $k = \frac{b_1}{2}$), which says that b_1 induces "sincere" voting cut-point $\frac{b_1}{2}$. In general, the strategic cut-point coincides with the sincere cut-point only in rare cases. When the rare events happen, it is possible that b_1 induces the same stochastic outcome in the straw poll. Nonetheless it is not sufficient. In addition to the condition that $b_1 = 2k^*$ (where k^* is the cut-point equilibrium in the straw poll), we also need to make sure that the setter's proposal $\widehat{b}_2(2)$ when she receives 2 endorsements is the same as b_1. It is not true in general so that we may not always be able to find one proposal in the binding institution that can induce the equilibrium lottery over policies in polling. However in some special cases it can happen. We propose the following technical condition, which is sufficient to guarantee that $2k^* = \widehat{b}_2(2; k^*)$.

Condition 1 suppose b_0 is the proposal of the setter with utility b without communication, providing $b_0 < 2G^{-1}(\frac{1}{2})$, where $G(\theta) = \lim_{\overline{\theta}\to+\infty} F(\theta;\overline{\theta})$.[2]

We summarize the results in the following proposition.

Proposition 8 *(1) If there exists any initial proposal in the binding referendum that can induce an equilibrium lottery over policies in polling with $k^* < \frac{1}{2}\theta_A$, it must be a compromising proposal inducing strategic sincere voting, i.e. $b_1 = 2k^*$, or the equilibrium cut-point $k^* = \frac{1}{2}b_1$.*

(2) Given Condition 1, as long as the preference divergence between the setter and voters is large, namely, $\exists M_1 > 0$,given any $\overline{\theta} \geq M_1$, $\exists M_2 \in (0, M_1)$ for any $\theta_A \in (M_2, \overline{\theta})$,

any equilibrium lottery over policies in the straw poll can always be obtained by setting $b_1 = 2k^* \in (b_0, \theta_A)$ under the binding vote, and the binding institution has an equilibrium that (weakly) dominates any equilibrium under the straw poll in terms of setter's welfare.

Proof *of* **Proposition** 8

We only need to show the second part, that is to show $b_0 < 2k$, so that $b_1 = 2k = b_2(0)$ can obtain lottery over policies in equilibrium of polling.

For large enough θ_A and $\overline{\theta}$, b_0 approximately equals to setter's optimal proposal without communication.

For large enough θ_A and $\overline{\theta}$, $F(\cdot)$ and $G(\cdot)$ are approximately the same, therefore $b_0 < 2G^{-1}(\frac{1}{2})$ implies $b_0 < 2F^{-1}(\frac{1}{2})$

If $2k \leq b_0$, we have $F(k) \leq F(\frac{1}{2}b_0) \leq \frac{1}{2}$, thus $(1 - F(k)) > F(k) - F(\frac{1}{2}b(1))$. It implies

$V_{diff}(k;k) = -(1 - F(k))[2kb_2(1)) - b_2(1)^2]$

[2] One can verify that when G follows exponential distribution $G(\theta) = 1 - \exp(-\lambda\theta)$, we have $b_0 < 2G^{-1}(\frac{1}{2})$. ($b_0 = \arg\max[1 - G(\frac{1}{2}x)]^2 x) = \frac{1}{\lambda} < \frac{2\ln 2}{\lambda} = 2G^{-1}(\frac{1}{2})$)

$+(F(k) - F(\frac{1}{2}b(1)))[2kb(1) - b(1)^2] - (F(k) - F(\frac{1}{2}b(2)))[2kb(0) - b(0)^2]$

< 0

which is a contradiction. As a result we must have $b_0 < 2k$ so that $b_1 = 2k = b_2(0)$ can obtain lottery over policies in equilibrium of polling.

Generalized Results

Proof *of* **Proposition** 6

According to Lemma 2, we only need to show that as long as θ_A is small enough, any equilibrium cut-point in the straw poll must be greater than $\frac{1}{2}\theta_A$.

Suppose it is not true, then we have at least one equilibrium cut-point $k < \frac{1}{2}\theta_A$ and

$\theta_A > b_2(n) > ... > b_2(q) \geq 2k$

$> b_2(q-1)... > b_2(1) > b_2(0).$

By the definition of the cut-point equilibrium, we must have $V_{diff}(\theta_i = \frac{1}{2}b_2(n); k) \geq 0$. In the following, we will show that it is not true.

In the following we try to show that for all j, when $\theta_i = \frac{1}{2}b_2(n)$,

$\binom{n-1}{j}F(k)^{n-j-1}[1 - F(k)]^j V(\theta_i, b_2(j+1), j)$

$< \binom{n-1}{j+1}F(k)^{n-j-2}[1 - F(k)]^{j+1} V(\theta_i, b_2(j+1), j+1)$

(1)

For $j \leq q - 1$,

$\because \theta_i = \frac{1}{2}b_2(n) \geq \frac{1}{2}b_2(j+1)$

$\therefore V(\theta_i, b_2(j+1), j+1) = [1 - \widetilde{F}_{n-2-j,n-q+1}(\frac{1}{2}b_2(j+1))][2\theta_i b_2(j+1) - b_2(j+1)^2] > 0$

and $V(\theta_i, b_2(j+1), j) = [1 - \widetilde{F}_{n-1-j,n-q+1}(\frac{1}{2}b_2(j+1))][2\theta_i b_2(j+1) - b_2(j+1)^2] > 0$

Based on Lemma 6, we have $\frac{1 - \widetilde{F}_{n-1-j,n-q+1}(\frac{1}{2}b_2(j+1))}{\widetilde{f}_{n-1-j,n-q+1}(\frac{1}{2}b_2(j+1))} < \frac{1 - \widetilde{F}_{n-2-j,n-q+1}(\frac{1}{2}b_2(j+1))}{\widetilde{f}_{n-2-j,n-q+1}(\frac{1}{2}b_2(j+1))}$.

Since we also have $G(\theta) = 1 - \exp(-\int_a^\theta \frac{g(s)}{1-G(s)} ds)$ for any distribution function $G(\theta)$,

$\widetilde{F}_{n-1-j,n-q+1}(\frac{1}{2}b_2(j+1)) > \widetilde{F}_{n-2-j,n-q+1}(\frac{1}{2}b_2(j+1))$

or

$[1 - \widetilde{F}_{n-2-j,n-q+1}(\frac{1}{2}b_2(j+1))] > [1 - \widetilde{F}_{n-1-j,n-q+1}(\frac{1}{2}b_2(j+1))]$

As a result,

$V(\theta_i, b_2(j+1), j+1) > V(\theta_i, b_2(j+1), j)$

Similarly, for $j \geq q$,

$\because \theta_i = \frac{1}{2}b_2(n) \geq \frac{1}{2}b_2(j+1)$

$\therefore V(\theta_i, b_2(j+1), j) = [1 - \widetilde{F}_{j,j-q+2}(\frac{1}{2}b_2(j+1))][2\theta_i b_2(j+1) - b_2(j+1)^2] \geq 0$

and $V(\theta_i, b_2(j+1), j+1) = [1 - \widetilde{F}_{j+1,j-q+3}(\frac{1}{2}b_2(j+1))][2\theta_i b_2(j+1) - b_2(j+1)^2] \geq 0$

Based on Lemma 6, we have $\frac{1-\widetilde{F}_{j,j-q+2}(\frac{1}{2}b_2(j+1))}{\widetilde{f}_{j,j-q+2}(\frac{1}{2}b_2(j+1))} < \frac{[1-\widetilde{F}_{j+1,j-q+3}(\frac{1}{2}b_2(j+1))]}{\widetilde{f}_{j+1,j-q+3}(\frac{1}{2}b_2(j+1))}$. Thus

$[1 - \widetilde{F}_{j,j-q+2}(\frac{1}{2}b_2(j+1))] < [1 - \widetilde{F}_{j+1,j-q+3}(\frac{1}{2}b_2(j+1))]$

So we have $V(\theta_i, b_2(j+1), j+1) > V(\theta_i, b_2(j+1), j)$.

(2)In the follow we show that

$\binom{n-1}{j+1}[1 - F(k)]^{j+1}F(k)^{n-2-j} >$

$\binom{n-1}{j}[1 - F(k)]^{j}F(k)^{n-1-j}$

which is equivalent to

$\frac{(n-1)!}{(n-j-2)!(j+1)!}(1 - F(k)) > \frac{(n-1)!}{(n-j-1)!j!}F(k)$

$\Leftrightarrow (n-1-j)(1 - F(k)) > (j+1)F(k)$

$\Leftrightarrow F(k) < \frac{n-j-1}{n}$

$\Leftarrow F(k) < \frac{1}{n}$ (because $j + 1 \leq n - 1$)

$\Leftarrow F(\frac{1}{2}\theta_A) < \frac{1}{n}$

(3)As long as the setter is sufficiently moderate, i.e. θ_A is very small, we must have $F(\frac{1}{2}\theta_A) < \frac{1}{n}$, which implies that $V_{diff}(\theta_i = \frac{1}{2}b_2(n); k) < 0$, which is a contradiction.

Proof *of* **Proposition** 7

The basic idea of the proof is to show the *lowest indifferent type* k must be weakly greater than $\frac{\theta_A}{2}$.

(1) Under simple majority rule, if $k < \frac{\theta_A}{2}$, let's check the incentive compatibility for types near k, i.e. $\theta_i \in (k - \epsilon, k + \epsilon)$.

(1.1) When the other voter is such that $\theta_j \geq k$,

If θ_i pretends to be the type above k, he will induce the setter to propose some $\bar{b} \geq 2k$ so that he gets $\beta(\bar{b})(2\theta_i\bar{b} - \bar{b}^2)$, where $\beta(\bar{b})$ is the probability the other voter will accept $\bar{b}$.

If θ_i pretends to be the type below k, he will induce the setter to propose some $\underline{b}$ so that he gets $\beta(\underline{b})(2\theta_i\underline{b} - \underline{b}^2)$.

By the same method in Lemma 4 and Lemma 5can show that $\bar{b} > \underline{b}$.

(1.2) When the other voter is such that $\theta_j < k$,

If θ_i pretends to be the type above k, he will make the setter believe the pivotal voter has an ideal point greater than k so that the induced proposal is weakly greater than $2k$. He knows the other voter will always rejects this proposal so that he is pivotal. If the proposal is strictly greater than $2k$, he will reject it and get 0. If the proposal equals to $2k$, he will get 0 anyway. As a result, in this case he always gets 0 payoff.

If θ_i pretends to be the type below k, he induces some proposal less than $2k$ so that he always accepts it and gets $2\theta_i b' - b'^2$,

where b' is the induced proposal and $b' < 2k$.

(1.3) The payoff gain (between signaling a type above k and signaling a type below k) is

$(1 - F(k))[\beta(\bar{b})(2\theta_i\bar{b} - \bar{b}^2) - \beta(\underline{b})(2\theta_i\underline{b} - \underline{b}^2) - F(k)[2\theta_i b' - b'^2]$

We must have

$$[1-F(k)][\beta(\overline{b})\overline{b}-\beta(\underline{b})\underline{b}]-F(k)b'>0 \tag{S7}$$

and

$$(1-F(k))[\beta(\overline{b})(2k\overline{b}-\overline{b}^2)-\beta(\underline{b})(2k\underline{b}-\underline{b}^2)]=F(k)[2kb'-b'^2] \tag{S8}$$

Inequality (S7) implies

$$\beta(\overline{b})\overline{b}-\beta(\underline{b})\underline{b}>0 \tag{S9}$$

Equation (S8) implies

$$\beta(\overline{b})(2k\overline{b}-\overline{b}^2)\geq\beta(\underline{b})(2k\underline{b}-\underline{b}^2) \tag{S10}$$

i.e.

$$k\geq\frac{\beta(\overline{b})\overline{b}^2-\beta(\underline{b})\underline{b}^2}{2[\beta(\overline{b})\overline{b}-\beta(\underline{b})\underline{b}]} \tag{S11}$$

We can verify that $\frac{\beta(\overline{b})\overline{b}^2-\beta(\underline{b})\underline{b}^2}{2[\beta(\overline{b})\overline{b}-\beta(\underline{b})\underline{b}]}>\frac{\overline{b}}{2}$ so that

$$k>\frac{\overline{b}}{2} \tag{S12}$$

which contradicts with $\overline{b}\geq 2k$. As a result, we have $k\geq\frac{\theta_A}{2}$.

(2) Under unanimity majority rule, if $k<\frac{\theta_A}{2}$, let's check the indifference condition for type $\theta_i=k$.

(2.1) When the other voter $\theta_j\geq k$,

If θ_i pretends to be the type above k, he will induce the setter to propose some policy greater than $2k$ so that he will always gets 0.

If θ_i pretends to be the type below k, he will induce the setter to propose some $b^* < 2k$ so that he gets $2\theta_i b^* - b^{*2}$.

(2.2) When the other voter $\theta_j < k$,

If θ_i pretends to be the type above k, he will induce the setter to propose b^* so that he will always gets $[1 - \frac{F(\frac{1}{2}b^*)}{F(k)}](2\theta_i b^* - b^{*2})$.

If θ_i pretends to be the type above k, he will induce the setter to propose some $\underline{b}' < 2k$ so that he will always gets $[1 - \frac{F(\frac{1}{2}\underline{b}')}{F(k)}](2\theta_i \underline{b}' - (\underline{b}')^2)$.

(2.3) We write down the indifference condition

$$-(1-F(k))(2kb^* - b^{*2}) + [F(k) - F(\frac{1}{2}b^*)](2kb^* - b^{*2}) = [F(k) - F(\frac{1}{2}\underline{b}')](2k\underline{b}' - (\underline{b}')^2) \tag{S13}$$

which requires $2F(k) - 1 - F(\frac{1}{2}b^*) > 0$ and $F(k) > \frac{1}{2}$.

Since $k < \frac{\theta_A}{2}$, we have $F(k) < F(\frac{\theta_A}{2})$. When $\theta_A \leq 2F^{-1}(\frac{1}{2})$ we get $F(k) < \frac{1}{2}$ which is a contradiction.

$2F(k) - 1 - F(\frac{1}{2}b^*) > 0$ implies $F(k) > \frac{1}{2}F(\frac{1}{2}b^*) + \frac{1}{2}F(\overline{\theta})$

When $F(\cdot)$ is convex, we have $F(\frac{\theta_A}{2}) > F(k) > F(\frac{1}{2}\frac{1}{2}b^* + \frac{1}{2}\overline{\theta})$, which is also a contradiction.

As a result, when $F(\cdot)$ is convex or $\theta_A \leq 2F^{-1}(\frac{1}{2})$, we always have $k \geq \frac{\theta_A}{2}$. Q.E.D.

Naive v.s. Sophisticated Voters in Straw Poll

In the following we compare the optimal cut-point (from the setter's point of view) and the equilibrium cut-point under the straw poll.

Under simple majority rule, when $k \geq \frac{1}{2}\theta_A$, setter's ex ante welfare can be written as

$E(u_A) = [1 - F^2(k)]\theta_A^2 + \underset{b_2}{Max}[F^2(k) - F^2(\frac{1}{2}b_2)][2\theta_A b_2 - b_2^2]$

By using Envelop Theorem, we have

$\frac{dE(u_A)}{dk} = -2F(k)f(k)\theta_A^2 + 2F(k)f(k)[2\theta_A b_2 - b_2^2]$

$= -2F(k)f(k)\{\theta_A^2 - 2\theta_A b_2 + b_2^2\}$

< 0

As a result[3], we have following proposition.

Proposition 9 *Suppose k^{**} is the cut-point which voters "naively" use to report their private information and k^* is the equilibrium cut-point. Under simple majority rule, we have $k^{**} < \frac{1}{2}\theta_A < k^*$.*[4]

This proposition suggests that it is optimal only if the cut-point is lower than the sincere type so that the setter can use compromise proposals to smooth the risk and improve welfare. By the definition of optimized polling, setter's welfare is higher under optimized straw poll than in any (symmetric) cut-point equilibrium.

[3]To make argument rigorous, we also need to verify that $\lim_{k \to \frac{1}{2}\theta_A} \frac{dE(u_A)}{dk} < 0$.

[4]This proposition holds also for unanimity rule, providing $F(\cdot)$ is convex or the setter is sufficiently moderate.