A Nonspeculation Theorem with an Application to Committee Design

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Abstract

Various well known agreement theorems show that if players have common knowledge of actions and a "veto" action is available to every player (Geanakoplos, 1994), then they cannot agree to forgo a Pareto optimal outcome simply because of private information in settings with unique equilibrium. We establish a nonspeculation theorem which is more general than Geanakoplos (1994) and is applicable to political and economic situations that generate multiple equilibria. We demonstrate an application of our result to the problem of designing an independent committee free of private persuasion.

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1 Introduction

In many situations from trade, to war, to committee decision making, reaching efficient outcomes often requires coordination. Complicating matters, in many of these circumstances players may have different preferences over which outcomes to coordinate on, private information regarding the value of the outcomes, and a veto action that allows them to secure a fixed outcome unilaterally. Regardless of the particulars of such "mutual acts," decision-makers in these situations face a similar strategic problem. The mutual act offers the possibility of higher rewards than not participating, but the higher benefit can only go to a subset of the participants because the veto outcome is Pareto optimal. When participants hold private information about the benefits of the mutual act, informal intuition suggests that the mutual act could occur due to "mutual optimism" or "agreeing to disagree"; because of their private information, both decision-makers could believe that they were likely to receive the high benefit of the mutual act.

Of course, it is well known from various agreement and no-trade theorems that such outcomes cannot arise from rational players that share some sort of common knowledge. The literature has shown that if players have a common prior and there is common knowledge of posteriors (Aumann, 1976), common knowledge of feasible trades (Milgrom and Stokey, 1982), or common knowledge of actions (Geanakoplos, 1994), then two players cannot agree to disagree or agree to forgo a Pareto optimal outcome simply because of private information. However, even this weakest condition, common knowledge of actions, seems like a strong requirement. As a theoretical matter, actions need not be common knowledge in a Bayesian Nash equilibrium and as a practical matter, common knowledge is unlikely to occur in decentralized trading systems or in settings like war where there is an incentive to disguise actions.

Taking this concern seriously, Geanakoplos (1994) proved the important but seemingly overlooked "Nonspeculation Theorem" that establishes that if all players in a Bayesian game have a veto action that results in a Pareto optimal outcome, then the veto outcome is the unique Bayesian Nash equilibrium outcome. The result suggests that as long as the power structure is appropriately designed, the efficient cooperative outcome can always be achieved independent of the information structure. It complements various agreement and no-trade theorems that show inefficient outcomes cannot arise from rational players that share some sort of common knowledge. By eliminating the need for players to possess common knowledge of actions, this weaker assumption broadens the scope of the no-trade literature to apply to Bayesian games with veto actions.

Though Geanakoplos (1994) provides an attractive result, it does not directly apply to many important political and economic environments where we usually have multiple equilibria (Myerson, 2013). In this paper, we present a Generalized Nonspeculation Theorem. Our extension of the Nonspeculation Theorem applies to situations where there are potentially many (equilibrium) outcomes, where there may be distributional consequences to different types of mutual acts, where coordination considerations are at play, and where the payoffs to all outcomes can be state dependent.[1]

As an application, we apply our theorem to the study of committee structures and show that as long as a "veto" power is endowed to every committee member, the members cannot agree to forgo the Pareto optimal outcome simply because of private information supplied by an interest group. This result contributes to recent literature on persuading committees with public information (Schnakenberg, 2015; Alonso and Câmara, 2014). Persuasion games have been widely studied in economics (Milgrom and Roberts, 1986; Gentzkow and Kamenica, 2011). Recently work has extended this approach to environments where receivers need to make a collective decision. For example, Schipper and Woo (2015) study how electoral campaigns can raise awareness of issues and unravel information asymmetries about candidates' policy positions. Schnakenberg (2015) shows that collegial voting rules (including unanimity rule) are free of manipulative public persuasion. Alonso and Câmara (2014) study how an information provider with commitment can persuade the committee to choose a particular policy.

The next section lays out the framework of Bayesian games that we consider. Section 3 presents the classical Geanakoplos theorem. Section 4 presents our main theorem. Section 5 applies the main result to a problem of designing an independent committee. Section 6 clarifies the conditions, and discusses our result. The final section concludes.

[1] Like Geanakoplos, we do not require common knowledge of posteriors, common knowledge of feasible trade, or common knowledge of actions; common knowledge of rationality is sufficient.

2 Speculation Games with Coordination

Suppose there are n players interacting by way of a finite strategic form game G. The set of *actions* for player i is given by a finite set A_i, with generic element a_i. As usual, an action profile $(a_1, \ldots, a_n)$ is denoted a, which is an element of $A = A_1 \times \cdots \times A_n$, and $a_{-i} = (a_1, \ldots, a_{i-1}, a_{i+1}, \ldots, a_n)$.

Let Ω be a finite set of *states*, where each $\omega \in \Omega$ describes a possible state of the world. We assume that the players share a *common prior* π over the state space. For simplicity, we assume that $\pi(\omega) > 0$ for all $\omega \in \Omega$. Let P_i be a partition of the state space Ω. As is standard, we interpret P_i as representing knowledge in the sense that for every event $E \subset \Omega$, if $P_i(\omega) \subseteq E$, then player i knows E has occurred at ω.

As the payoffs of the game can depend on the state of world, $u_i(a, \omega)$ denotes the utility of player i when the action a is played in state ω. We now define *strategies* for each player. We reflect the fact that players can condition their choice of action on their private information by defining a (mixed) strategy s_i as a function $s_i : \Omega \to \Delta A_i$ with the restriction that

$$P_i(\omega) = P_i(\omega') \quad \Rightarrow \quad s_i(\omega) = s_i(\omega').$$

This condition states that if a player cannot distinguish state ω from state ω', then her action distribution must be the same in both states. The set of all strategies for player i is denoted S_i. A profile of strategies $(s_1, \ldots, s_n)$ is denoted s, which is an element of $S = S_1 \times \cdots \times S_n$, and s_{-i} refers to the strategy profile of all players except player i. With a slight abuse of notation, we use $u_i(s(\omega), \omega)$ to denote player i's expected payoff conditional on ω given that the profile strategy is $s(\omega)$. The expected utility of strategy profile s to player i conditional on the event $D \subseteq \Omega$ is

$$E[u_i(s(\omega), \omega)|D] = \frac{\sum_{\omega' \in D} u_i(s(\omega'), \omega')\pi(\omega')}{\sum_{\omega' \in D} \pi(\omega')}.$$

Sometime we write it as $E[u_i(s, \omega)|D]$ for simplicity. Notice that $E[u_i(s(\omega), \omega)|D]$ is calculated by the expectation operator so that it should not depend on the state ω. In particular, the ex ante payoff of player i from strategy profile s is given by $E[u_i(s, \omega)|\Omega]$. The expected utility of strategy profile s to player i with information $P_i(\omega')$ at state ω' is given by $E[u_i(s, \omega)|P_i(\omega')]$.

A *Bayesian Nash equilibrium* is a strategy profile s^* such that for all i, for all $a_i \in A_i$, and for all $\omega' \in \Omega$,

$$E[u_i(s^*(\omega), \omega)|P_i(\omega')] \geq E[u_i(a_i, s^*_{-i}(\omega), \omega)|P_i(\omega')].$$

This definition means that for every possible piece of private information $P_i(\omega)$, player i's equilibrium action is optimal, given the equilibrium strategies of the other players.[2]

3 Geanakoplos' Nonspeculation Theorem

In this part, we present the Nonspeculation Theorem of Geanakoplos (1994). The theorem concerns Bayesian games in which each player has an ex ante Pareto optimal veto action. It establishes that every such game has a unique Bayesian Nash equilibrium in which all players play their veto action.

To begin, we give the formal description of the Pareto optimal veto action condition of Geanakoplos.

Condition 1 (Geanakoplos) *For each player i, there exists an action $z_i \in A_i$ such that for all s, $E[u_i(z_i, s_{-i}(\omega), \omega)|\Omega] = v_i$, and if s satisfies $E[u_i(s(\omega), \omega)|\Omega] \geq v_i$ for all i, then for all j, $s_j(\omega) = z_j$ for all $\omega \in \Omega$.*

The first part of this condition requires that each player i have a veto action z_i. This action is a veto action because it assures player i a fixed payoff (of v_i), no matter what actions the other players play. The second part of this condition requires that the veto outcome is Pareto optimal: if every player is receiving at least their veto action payoff, then all players must be playing their veto action.

We can now state the Nonspeculation Theorem.

Theorem 1 (Geanakoplos) *If a Bayesian game satisfies Condition 1, then it has a unique Bayesian Nash equilibrium s^*, where $s^*_j(\omega) = z_j$ for all $\omega \in \Omega$ and for all j.*

[2] Following Geanakoplos (1994) and others in the literature, we study the standard Bayesian Nash equilibrium. But if we were to use a weaker solution concept, like BNE with non-common priors or *interim correlated rationalizibility* introduced in Dekel, Fudenberg and Morris (2007), speculative trade would be possible.

		Player 2	
		Trade	No Trade
Player 1	Trade	$-2, -2$	$-c, 0$
	No Trade	$0, -c$	$0, 0$

Figure 1. A Trading Game

It is not hard to see the logic of the theorem. Take a strategy profile in which $s_j(\omega) \neq z_j$ for some $\omega \in \Omega$ and some j. By the second part of Condition 1, there exists a player i whose ex ante utility is strictly less than v_i. But then by the first part of Condition 1, player i can achieve a strictly higher ex ante payoff by playing z_i in all states. It follows that there is some partition element $P_i(\omega')$ such that playing $s_i(\omega')$ is strictly worse than playing z_i, but this means the strategy profile s is not a Bayesian Nash equilibrium. It follows that the unique Bayesian Nash equilibrium of the game is given by $s_j^*(\omega) = z_j$ for all $\omega \in \Omega$ and for all j. This result suggests that more information cannot change the collective outcome from the ex ante efficient status quo, as long as all players have veto power to maintain the status quo.

It should be noted that although the uniqueness result in the Nonspeculation Theorem is very strong, it is due to the fact that Condition 1 is also quite strong. This condition requires that if every player is receiving at least their veto action payoff, then *every* player must be playing their veto action.

To see why this is strong, consider a standard bilateral trade game (Figure 1) where two players must decide whether to agree to a trade of some asset. In this game, there is no uncertainty and trade is voluntary in that both players must agree in order for the trade to be completed. We suppose that a completed trade makes both players worse off than not trading and we allow there to be a cost c of offering a trade that is rejected by the other player. Clearly, both sides have a veto action, with payoff 0.

It is obvious in this simple game that there will be no trade in equilibrium, but if $c = 0$ (so that players care only about whether or not trade occurs), then Condition 1 is not satisfied. For the strategy profile (Trade, No Trade), for example, both players are receiving their veto action payoff, but player 2 is not choosing her veto action. Thus the Nonspeculation Theorem does not apply to this game. A related concern is that in this trade game there are multiple equilibria, which cannot happen under the Nonspeculation Theorem.

On the other hand, the case of $c > 0$ illustrates a game in which Condition 1 and the Nonspeculation Theorem do apply. If $c > 0$ and so offering a trade that is rejected is costly, it is clear that the only way for either player to achieve the veto action payoff of 0 is to choose the veto action. This satisfies Condition 1 and, as required by the Nonspeculation Theorem, there is a unique equilibrium in which both players choose their veto action.

4 Main Result

In order to deal with a wider range of situations, we propose a new condition to replace Condition 1. First, denote *a common refinement* of the partitions $P_1, \ldots, P_n$ by a partition $\widehat{P}$. That means, every element of $\widehat{P}$ is a subset of some element of P_i, for all i. A special example of a common refinement is the coarsest common refinement of the partitions, namely the join of the partitions $P_1, \ldots, P_n$, which we denote by the partition P^*.[3] In terms of knowledge, the join of the possibility correspondences of the players represents what players would know if their information were public instead of private. This is what Fagin et al. (2003) call "distributed knowledge" and represents what would be known if everyone truthfully shared their private information. Another special example of a common refinement is the finest common refinement, the elements of which are all singletons. Now consider Condition 2.

Condition 2 *For each player i, there exists an action $z_i \in A_i$ such that for all s, $E[u_i(z_i, s_{-i}(\omega), \omega)|\Omega] = v_i$, and if s satisfies $E[u_i(s(\omega), \omega)|\Omega] \geq v_i$ for all i, then for all j, $E[u_j(s(\omega), \omega)|\widehat{P}(\omega')] = E[u_j(z, \omega)|\widehat{P}(\omega')]$ for all $\omega' \in \Omega$, where partition $\widehat{P}$ is a common refinement of the partitions $P_1, \ldots, P_n$.*

Here, the only difference from the earlier condition is that in Condition 2, if a strategy profile s makes everyone ex ante weakly better off than in the veto outcome, s does not necessarily need to be the veto strategy profile. Instead, we only require that s gives the same expected payoffs as the veto outcome from the

[3] For example, suppose the partitional structure of player 1 is $\{\{\omega_1, \omega_2\}, \{\omega_3, \omega_4\}, \{\omega_5, \omega_6, \omega_7\}\}$, the partitional structure of player 2 is $\{\{\omega_1\}, \{\omega_2, \omega_3\}, \{\omega_4, \omega_5, \omega_6\}, \{\omega_7\}\}$. Thus the join of the two partitions is $\{\{\omega_1\}, \{\omega_2\}, \{\omega_3\}, \{\omega_4\}, \{\omega_5, \omega_6\}, \{\omega_7\}\}$. Another common refinment is $\{\{\omega_1\}, \{\omega_2\}, \{\omega_3\}, \{\omega_4\}, \{\omega_5\}, \{\omega_6\}, \{\omega_7\}\}$, which is finer than the join.

		Player 2		
	ω_1	T	N^1	N^2
	T	$-2,-2$	$0,0$	$0,0$
Player 1	N^1	$0,0$	$1,1$	$0,0$
	N^2	$0,0$	$0,0$	$0,0$

		Player 2		
	ω_2	T	N^1	N^2
	T	$-2,-2$	$0,0$	$0,0$
Player 1	N^1	$0,0$	$-1,-1$	$0,0$
	N^2	$0,0$	$0,0$	$0,0$

Figure 2. A Trading Game

perspective based on a common refinement partition $\widehat{P}$. In this way, we allow for multiple efficient strategy profiles and multiple equilibria.

Note as well that the payoffs of s are equal to the payoffs of the veto action for every element of a common refinement partition $\widehat{P}$. In the simple case in which every element of $\widehat{P}$ is a singleton, this part of the condition reduces to $u_j(s(\omega), \omega) = u_j(z, \omega)$ for all j and all ω. That is, s and z have the same ex post payoff. In the general case, however, we only require that s and z have the same expected payoff across each element of $\widehat{P}$.

As an example of this, consider Figure 2, which is a modification of the bilateral trade game given earlier. Here, $\Omega = \{\omega_1, \omega_2\}$ and we suppose that $\pi(\omega_1) = \pi(\omega_2) = 1/2$ and $P_1 = P_2 = \Omega$. Thus, neither player has private information about the state of the world and therefore $P^* = \Omega$. Clearly, N^2 is a veto action for both players. In addition, though, N^1 gives each player the same expected payoff relative to P^* as N^2 and should also be covered by our condition, even though the payoffs differ in state ω_1 and ω_2. In this way, our condition covers more games and information structure than a strictly ex post formulation. [4]

It should also be clear that Condition 2 is a generalization of Condition 1. Whenever Condition 1 is satisfied, we have $s(\omega) = z$ for all ω and therefore Condition 2 is automatically satisfied.

Under the new condition, we generalize the theorem as follows.

Theorem 2 *If a Bayesian game satisfies Condition 2, then every Bayesian Nash equilibrium s^* induces the same expected payoffs as the efficient veto outcome (measurable with respect to the common refinement partition $\widehat{P}$). That is, $E[u_i(s^*(\omega), \omega)|\widehat{P}(\omega')] = E[u_i(z, \omega)|\widehat{P}(\omega')]$ for all $\omega' \in \Omega$ and all i.*

[4] As an alternative, we could modify our condition to require s have the same ex ante payoff as z. In this case, the equilibrium payoffs in the following theorem can be guaranteed to be the same as in the veto outcome only from the ex ante point of view.

Proof *of* **Theorem 2:** By the definition of Bayesian Nash Equilibrium, we have

$$E[u_i(s^*(\omega), \omega)|P_i(\omega')] \geq E[u_i(z_i, s^*_{-i}(\omega), \omega)|P_i(\omega')],$$

for all $\omega' \in \Omega$ and all i. From this it follows that

$$E[u_i(s^*(\omega), \omega)|\Omega] \geq v_i,$$

for all i. It follows immediately from Condition 2 that

$$E[u_i(s^*(\omega), \omega)|\widehat{P}(\omega')] = E[u_i(z, \omega)|\widehat{P}(\omega')],$$

for all $\omega' \in \Omega$ and all i. ■

By relaxing Geanakoplos' original condition, now the new theorem can be applied to political environments where multiple equilibria arise. In the following section, we use an example of committee design to illustrate the application of the theorem.

5 An Application: Designing the Power Structure of a Committee

As an application of our result, we study a problem of designing an independent committee when an interest group, as an information provider, can devise private signals to influence committee members' collective decisions.

Specifically, we assume that there is a committee of n members and each committee member i makes an individual decision $a_i \in A_i$. Each profile of decisions $(a_1, \ldots, a_n)$ generates a public policy $f(a_1, \ldots, a_n) \in X$, and for every policy $x \in X$, there exists a profile of decisions $(a_1, \ldots, a_n)$ such that $x = f(a_1, \ldots, a_n)$. The payoff of committee member i, $u_i(x, \omega)$, depends directly on the policy outcome as well as the state of the world ω. As before, the set of possible states of the world is given by Ω and nature draws a state of the world from a common prior π.

5.1 Private Information and Committee Decision

Consider the case where each committee member has an exogenous information partition $\{P_i(\cdot)\}$ of Ω that represents a committee member's private information. Having observed the element of the partition P_i that contains the true state, each committee member i simultaneously makes their individual decision a_i. As in the benchmark, players can use mixed strategies those are potentially state dependent.

Suppose a socially efficient policy exists.

Assumption 1 (Existence of An Efficient Policy) *There is a policy $Q \in X$ and values $\lambda_i > 0$ for $i = 1, \dots, n$ such that*

$$\sum_i \lambda_i u_i(x, \omega) < \sum_i \lambda_i u_i(Q, \omega),$$

for every $\omega \in \Omega$ and for every $x \neq Q$.

Our question is how to design the power structure in the committee, i.e., the policy function $f(a_1, \dots, a_n)$ together with the choice set $A = A_1 \times \cdots \times A_n$, such that the efficient policy Q will always be implemented for all possible information structures $\{P_1, \dots, P_n\}$. Substantively, we can think of a story where the information provider (i.e., a lobbyist) wants to persuade the committee to implement a policy other than the efficient one by her choice of the information structure $\{P_1, \dots, P_n\}$. Here we not only allow public persuasion via a public signal, such as in Schnakenberg (2015) and Alonso and Câmara (2014), we also allow the possibility of private persuasion.

We propose the following form of power as a sufficient solution.

Assumption 2 (Veto Power) *The power structure $\{f(a_1, \dots, a_n), A_1 \times \cdots \times A_n\}$ gives a "veto" power to every committee member, that is, for each player i, there exists an action $z_i \in A_i$ such that for all $a_{-i} \in A_{-i}$, $f(z_i, a_{-i}) = Q$.*

As a direct corollary of Theorem 2, we have

Proposition 1 *If Assumptions 1 and 2 hold, then for every information structure $\{P_1, \dots, P_n\}$, the committee decision induces the same ex post payoffs as the efficient policy Q.*[5]

[5] In fact, we can show a stronger result that the efficient policy Q will be implemented with probability 1.

Proof *of* **Proposition 1:** Fix an arbitrary information structure $\{P_1, \ldots, P_n\}$. To begin, for each $i = 1, \ldots, n$, let $v_i = E[u_i(Q, \omega)|\Omega]$. By Assumption 2 it is immediate that $E[u_i(f(z_i, s_{-i}(\omega)), \omega)|\Omega] = v_i$ for all s and so the first part of Condition 2 is satisfied. For the second part, suppose s satisfies $E[u_i(f(s(\omega)), \omega)|\Omega] \geq v_i$ for all i. Under this strategy s, policy Q must be implemented with probability 1, as otherwise $\sum_i \lambda_i E[u_i(f(s(\omega)), \omega)|\Omega] < \sum_i \lambda_i v_i$ by Assumption 1, which contradicts the fact that $E[u_i(f(s(\omega)), \omega)|\Omega] \geq v_i$ for all i. As a result, we must have $u_i(f(s(\omega)), \omega) = u_i(Q, \omega)$ for all ω, so that the second part of Condition 2 is satisfied with respect to the finest common partition, the elements of which are all singletons. Now, applying Theorem 2, we conclude that every equilibrium has the same ex post payoffs as the efficient policy Q, i.e., $u_i(f(s^*(\omega)), \omega) = u_i(Q, \omega)$ for all $\omega' \in \Omega$ and all i. ■

5.2 When there is an information provider

The proposition above shows that the collective decision made by a committee in which all members have veto power gives members the same payoffs as the efficient status quo. Most importantly, the equilibrium payoffs are independent of their private information. Hence, even if there is an information provider who can endogenously affect the private belief of each committee member, the equilibrium payoffs remain unaffected. For example, suppose that an information provider, such as a lobbyist, is able to choose how private information is supplied to the committee members. We formalize this informational lobbying example with the following timing:

Stage 1 Each member i privately observes the signal induced by the partition P_i;

Stage 2 The information provider who knows the true state ω supplies a profile of information partitions $\{P_i^S\}_{i=1}^n$, and the signal induced by the partition $P_i^S(\omega)$ is privately observed by i;

Stage 3 Members simultaneously make their individual decisions $a_1, a_2, \ldots, a_n$.

In this specific game, no matter what we assume about the information provider, for example if she has a commitment power or not[6], the information available to

[6] For the information provider with no commitment, when we pin down the equilibrium strategy of $\{P_i^S\}$, we need to require the incentive compatibility constraints be satisfied.

each committee member i in equilibrium just before she makes a decision a_i is represented by a combination of two sources of information. One piece of information is her original private information, which is represented by the partition P_i. The second source of information is from the information provider, and is represented by the partition P_i^S. Therefore, her updated information just before she makes the decision is represented by the join of the two partitions. Since both sources of signals are partitional, the join of them is partitional. Then by directly applying Proposition 1, we know that the equilibrium committee decision induces the same ex post payoffs as the efficient status quo policy $Q$is.

Notice that the timing we describe above is similar to the game structure in Schnakenberg (2015) (where the information provider has no commitment) and Alonso and Câmara (2014) (where the information provider has a commitment). The main feature in our framework is that we allow the information provider to privately persuade each committee member so that the message received by each committee member may be different, whereas Schnakenberg (2015) and Alonso and Câmara (2014) consider the effect of public signals on collective decisions.

5.3 Institutional Arrangements with Veto Power

To better understand the role of veto power in our condition, consider the following concrete example.

Suppose there are two options: the reform policy R, and the status quo policy Q, so that $X = \{Q, R\}$ and $A_i = \{Y, N\}$ for $i = 1, \ldots, n$. We restrict our attention to quota rules with quota q. Specifically in the decision process, each committee member i casts a vote $a_i \in \{Y, N\}$ and the reform policy R wins if and only if q or more members vote Yes, for some fixed $q \in \{1, \ldots, n\}$.

In this example, Assumption 2 is equivalent to a unanimity requirement, i.e., $q = n$. Therefore Proposition 1 shows that unanimity rule guarantees an efficient collective decision when the status quo is more efficient than reform, regardless of the private persuasion chosen by the information provider.[7]

Now, suppose there are more than two options, namely $X = \{R_1, R_2, ..., R_k, Q\}$, with $k > 2$. We can think of a more general example in which the institutional

[7]With other voting rules ($q < n$), there exists an non-empty open set of committee members' preferences (that are consistent with Assumption 1), under which the probability that the equilibrium outcome is efficient is strictly less than 1 (even if we exclude weakly dominated strategies).

arrangement is asymmetric with respect to different policies. Each committee member i can make a policy proposal a_i chosen from all possible policies, i.e., $a_i \in A_i = \{R_1, R_2, ...R_k, Q\}$. In this example, Assumption 2 is equivalent to an institutional power structure such that the policy Q will be implemented whenever there is at least one member who proposes it (i.e., $f(a_1, a_2, ..., a_n) = Q$, if $a_i = Q$ for some i.). We do not need to make any further assumption about the proposal aggregation function $f(a_1, a_2, ..., a_n)$ when none of the members proposes the status quo policy Q. As long as everyone has freedom to propose any feasible policy, Proposition 1 suggests that such an institutional arrangement guarantees an efficient collective decision when the status quo policy is the most efficient policy choice, regardless of informational persuasion by the information provider.

6 Discussions

6.1 A Practical Condition

Although the new theorem is more general, its assumptions, especially the second part of Condition 2, may be difficult to verify especially because we need to check the requirement for all possible mixed strategies, mapping states into mixtures over actions. Thus, we formulate the following condition as a practical replacement which can be verified by checking profiles of actions only.

Condition 3 *For each player i, there exists an action $z_i \in A_i$ and a function $v_i : \Omega \to \mathbb{R}$ such that for all $a \in A$ and all $\omega' \in \Omega$, $E[u_j(z_i, a_{-i}, \omega)|\widehat{P}(\omega')] = v_j(\omega')$ for all j, where $\widehat{P}$ is a common refinement partition of $P_1, \ldots, P_n$, and for all $a \in A$ either $E[u_i(a, \omega)|\widehat{P}(\omega')] = v_i(\omega')$ for all i and all $\omega' \in \Omega$ or $\sum_i E[u_i(a, \omega)|\widehat{P}(\omega')] < \sum_i v_i(\omega')$ for all $\omega' \in \Omega$.*

The first part of this condition requires that each player i have a veto action z_i. This veto action gives every player their veto-outcome payoff $v_j(\omega)$, which by construction is measurable with respect to $\widehat{P}$. The second part of the condition is the social efficiency condition: at every element of the common refinement $\widehat{P}$, the veto outcome makes everyone weakly better than any other outcome.

We next verify that Condition 3 is sufficient for Condition 2 to hold.

Theorem 3 *Condition 3 implies Condition 2.*

This result is proved in the Appendix. Obviously, the importance of this theorem is that it means Condition 3 is sufficient for the conclusion of Theorem 2 to hold.

As an example of this condition, recall the second example in the last section. Suppose that the status quo policy Q is socially efficient when all player have equal weights. Then the members' actions a either give the policy Q, so that $u_i(x,\omega) = u_i(Q,\omega)$ for all i and all ω or give a policy $x \neq Q$, so that $\sum_i u_i(x,\omega) < \sum_i u_i(Q,\omega)$ for all $\omega' \in \Omega$. This allows us to easily establish that Condition 3 is satisfied and therefore the conclusions of Theorem 2 hold for this example.

6.2 The Role of Social Efficiency

A basic assumption of the main theorem and in the no-trade literature in general is that the veto outcome is socially efficient when conditioning on the common refinement. But with multiple possible outcomes and general game forms, it is natural to wonder what can happen when this assumption is violated. In this section we give two brief examples that illustrate the role of social efficiency for our result.

The distinction that we highlight relates to the conclusion of Theorem 2. According to our result, if a Bayesian game satisfies Condition 2, then the equilibrium payoff is always socially efficient, relative to the common refinement partition $\widehat{P}$, which we specify in Condition 2. However, if a game fails to satisfy the condition because there are a number of outcomes, some of which are socially preferable to the veto outcome, then socially inefficient outcomes (as measured relative to $\widehat{P}$) can occur with positive probability in equilibrium. In other words, the possibility of socially superior outcomes can give rise to the risk of social inefficiency in equilibrium.

Again, it is important to emphasize that our results are relative to the common refinement partition $\widehat{P}$, which we specify in Condition 2. In fact, even if a game satisfies Condition 2, it is possible that a socially inefficient outcome can occur at some specific state. For example, in the game given in Figure 2, the profile (N^1, N^1) is an equilibrium but the payoff to this strategy in state ω_2 is socially inefficient. However, it is clear that the payoff to this strategy is socially efficient relative to the specified common refinement partition $\widehat{P} = \Omega$.

For our first example, consider the game illustrated in Figure 3. Obviously, the

		Player 2		
		b_1	b_2	$\tilde{b}$
	a_1	$2,2$	$-1,-1$	$0,0$
Player 1	a_2	$-1,-1$	$2,2$	$0,0$
	$\tilde{a}$	$0,0$	$0,0$	$0,0$

Figure 3. A Trading Game with Mutually Beneficial Trade

		Player 2		
	ω_1	b_1	b_2	$\tilde{b}$
	a_1	$-1,-1$	$0,0$	$0,0$
Player 1	a_2	$1,5$	$1,1$	$0,0$
	$\tilde{a}$	$0,0$	$0,0$	$0,0$

		Player 2		
	ω_2	b_1	b_2	$\tilde{b}$
	a_1	$-1,-1$	$-1,-1$	$0,0$
Player 1	a_2	$-1,-1$	$-1,-1$	$0,0$
	$\tilde{a}$	$0,0$	$0,0$	$0,0$

		Player 2		
	ω_3	b_1	b_2	$\tilde{b}$
	a_1	$-1,-1$	$5,1$	$0,0$
Player 1	a_2	$0,0$	$1,1$	$0,0$
	$\tilde{a}$	$0,0$	$0,0$	$0,0$

Figure 4. A Game with Private Information

two outcomes with payoffs (2, 2) socially dominate the veto outcome with payoff (0, 0). Assume that $\widehat{P} = \Omega = \{\omega\}$ so there is no private information. It is clear that this game has a mixed strategy equilibrium in which both players play their first two strategies with probability 1/2. In this equilibrium, an outcome that is socially worse than the veto outcome occurs with probability 1/2.

Our second example makes a similar point but with private information and avoiding mixed strategies. Suppose there are three states of the world, ω_1, ω_2, and ω_3 each corresponding to one of the normal form games shown in Figure 4. Note that the outcome corresponding to the action pair (a_1, b_1) is socially worse than the veto outcome in every state of the world and in state ω_2 every outcome is socially worse than the veto outcome. Suppose the common prior probabilities on the states are $\pi(\omega_1) = \pi(\omega_2) = \pi(\omega_3) = 1/3$. Also assume that the players' information structure is

$$P_1 : \{\omega_1\}\{\omega_2, \omega_3\}$$
$$P_2 : \{\omega_1, \omega_2\}\{\omega_3\}.$$

As a consequence, we have that $P^* = \{\{\omega_1\}, \{\omega_2\}, \{\omega_3\}\}$. We focus on $\widehat{P} = P^*$.

Consider the following strategy profile: $s_1(\omega_1) = a_2$ and $s_1(\omega_2) = s_1(\omega_3) = a_1$, and $s_2(\omega_1) = s_2(\omega_2) = b_1$ and $s_2(\omega_3) = b_2$. It is easy to check that this strategy profile is a Bayesian Nash equilibrium and in this equilibrium, the socially inferior outcome corresponding to the action pair (a_1, b_1) occurs at state ω_2. Here, then, we get a socially inefficient outcome with probability 1/3.

These two examples illustrate the importance of social efficiency of the veto outcome in a setting with multiple possible outcomes. In both of these examples, the ex ante value of the equilibrium to both players is higher than the veto outcome, but this occurs at the risk of socially inefficient outcomes occurring. However, there is no risk of this occurring in games in which the veto outcome is efficient.

7 Conclusions

In this paper we have presented an extended version of Nonspeculation Theorem to show that equilibrium play cannot lead to a socially inefficient outcome in games with private information. Our version can be more broadly applied to political environments with multiple equilibria. Our result suggests that mutual optimism due to private information cannot be a cause of suboptimal outcomes such as inefficient trade or costly war.

As an application, we also use our theorem to study a problem of committee design when an interest group can provide private signals to influence committee members' decisions. We show that as long as a "veto" power is endowed to every committee member, they cannot agree to forgo the Pareto optimal outcome simply because of private information. This result complements recent literature of persuading committee with public information (Schnakenberg, 2015; Alonso and Câmara, 2014) by expanding the set of possible information structures and allowing private persuasion.

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Appendix

Proof *of* **Theorem 3:** Let $s^a(\omega)$ denote the probability with which an action profile a is played when the strategy profile is $s(\omega)$, and the state is ω.

Let's first establish a useful identity. For any strategy profile $s(\cdot)$ and any ω', we have

$$\begin{aligned}
E[u_i(s(\omega),\omega)|\omega \in \widehat{P}(\omega')] &= E[\sum_{a\in A} u_i(a,\omega)s^a(\omega)|\omega \in \widehat{P}(\omega')] \\
&= \sum_{a\in A} E[u_i(a,\omega)s^a(\omega')|\omega \in \widehat{P}(\omega')] \\
&= \sum_{a\in A} E[u_i(a,\omega)|\omega \in \widehat{P}(\omega')]s^a(\omega')
\end{aligned}$$

(1) Assume that Condition 3 holds and for $i = 1,\ldots,n$, let $v_i = \sum_{\omega\in\Omega} \pi(\omega)v_i(\omega)$. For all s and for all i, we have

$$\begin{aligned}
E[u_i(z_i, s_{-i}(\omega),\omega)|\Omega] &= \sum_{\omega'\in\Omega} \pi(\omega')E[u_i(z_i, s_{-i}(\omega),\omega)|\widehat{P}(\omega')] \\
&= \sum_{\omega'\in\Omega} \pi(\omega') \sum_{a_{-i}} E[u_i(z_i, a_{-i},\omega)|\widehat{P}(\omega')]s_{-i}^{a_{-i}}(\omega') \\
&= \sum_{\omega'\in\Omega} \pi(\omega') \sum_{a_{-i}} v_i(\omega')s_{-i}^{a_{-i}}(\omega') \\
&= \sum_{\omega'\in\Omega} \pi(\omega')v_i(\omega') \\
&= v_i
\end{aligned}$$

This establishes the first part of Condition 2.

(2) For the second part of Condition 2, suppose s satisfies $E[u_i(s(\omega),\omega)|\Omega] \geq v_i$

for all i. This is equivalent to

$$\sum_{\omega' \in \Omega} \pi(\omega') E[u_i(s(\omega), \omega) | \widehat{P}(\omega')] \geq v_i$$

for all i. Summing across individuals, we have

$$\sum_{\omega' \in \Omega} \pi(\omega') \sum_i E[u_i(s(\omega), \omega) | \widehat{P}(\omega')] \geq \sum_i v_i.$$

By the second part of Condition 3, we have two types of action profiles: the ones such that $E[u_i(a, \omega) | \widehat{P}(\omega')] = v_i(\omega')$ for all i and all $\omega' \in \Omega$; and the action profiles such that $\sum_i E[u_i(a, \omega) | \widehat{P}(\omega')] < \sum_i v_i(\omega')$ for all $\omega' \in \Omega$.

Given a strategy profile $s(\cdot)$, for any state ω', if the second types of actions are never played with a positive probability, then according to the identity we show above, we get $E[u_i(s(\omega), \omega) | \widehat{P}(\omega')] = \sum_{a \in A} v_i(\omega') s^a(\omega') = v_i(\omega')$, for all i. If this property is satisfied for all states ω', then we get the result we want.

If, however, under some state ω', the second types of action are played with a positive probability, we have

$$\begin{aligned}
\sum_i E[u_i(s(\omega), \omega) | \widehat{P}(\omega')] &= \sum_i \sum_{a \in A} E[u_i(a, \omega) | \widehat{P}(\omega')] s^a(\omega') \\
&= \sum_{a \in A} \sum_i E[u_i(a, \omega) | \widehat{P}(\omega')] s^a(\omega') \\
&< \sum_{a \in A} \sum_i v_i(\omega') s^a(\omega') \\
&= \sum_i v_i(\omega').
\end{aligned}$$

Summing across all possible states, we have $\sum_{\omega' \in \Omega} \pi(\omega') \sum_i E[u_i(s(\omega), \omega) | \widehat{P}(\omega')] < \sum_{\omega' \in \Omega} \pi(\omega') \sum_i v_i(\omega') = \sum_i v_i$.

This is a contradiction and thus we have $E[u_j(s(\omega), \omega)|\widehat{P}(\omega')] = E[u_j(z, \omega)|\widehat{P}(\omega')]$ for all j and all ω'. This establishes the second part of Condition 2.

■